Rational Choice and the Rule against Perpetuities

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The common-law Rule against Perpetuities disallows bequests to unborn distant generations. The Rule reflects social concern over the potential consequences of property restrictions imposed by long-deceased former owners. This paper considers the implications of rational choice for the welfare properties of the Rule. Unfettered bequests can be used to balance the competing interests of subsequent generations of beneficiaries. The alternative of disallowing distant bequests gives greater control to early generations of beneficiaries, permitting them to favor their own interests at the expense of future generations. As a result, the Rule against Perpetuities typically reduces dynastic welfare if intergenerational transfers are rationally, even selfishly, based, suggesting that the case favoring the Rule must rely on the potential for irrational behavior.

JEL Classifications: K11, D91.
1. Introduction

The Rule against Perpetuities (“the Rule”) is a common-law prohibition against distant vesting of contingent interests. John Chipman Grey (1906, p. 166) offered a classic formulation of the Rule, which is that an interest is valid only if it is certain to vest within the span of a life in being plus 21 years. A settlor attempting to establish a trust that pays income to surviving children for life, followed by paying income to their surviving children for life, followed by their children’s children, and so on forever down the generations, would run afoul of the Rule and have the distant future interests in the trust ruled invalid. Perpetuities, indeed, need not be implicated: despite its name, the Rule invalidates non-perpetual interests that are not certain to vest within the allotted time period. An important practical implication of the Rule is that it is not possible to transfer resources to sufficiently distant future generations; instead, any attempted transfer must pass through intermediate generations that may decide to consume or redirect them, possibly leaving little or nothing to future generations.

The Rule against Perpetuities is controversial. Its origins lie in cases of attempted contingent transfers that spanned generations and that too clearly attempted to exert dead hand control over the affairs of people unknown to the transferors. Once adopted, the Rule was also commonly justified by the inefficiency that can accompany unforeseen circumstances in the distant future. The creation of perpetual or very distant interests can prevent, or make quite costly, efficiency-enhancing property reallocations, such as physical improvements or exchanges that would benefit all parties. But more importantly, supporters of the Rule doubt the value of adhering to the wishes and whims of long-dead one-time property owners, whether or not the resulting allocations are efficient; and virtually all observers concede the practical impossibility of anticipating all future contingencies at the time that perpetual or distant future interests are created. Supporters argue that in the absence of the Rule, society would enforce the often imprudent, unwise, silly, or contrary to intention terms of long-distant instructions.1

In contrast, opponents of the Rule see its restrictions as improper intrusions on private property rights. They ask why distant vesting of contingent interests is disallowed while other, arguably less noble, forms of property disposition are permitted. During their lifetimes, owners

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1 See, for example, Simes (1955a, b).
are largely free to squander their fortunes without legal interference. Opponents point out that, in addition to its limitation on the range of dispositive options, the Rule represents a technicality of which many can run afoul. The Rule’s restrictions on distant vesting may trap the unwary or those with imperfect legal advice, throwing into disarray succession plans, many of which realistically were unlikely even to entail distant vesting.\(^2\)

The purpose of this paper is to consider the role of rational behavior in evaluating the desirability of the Rule against Perpetuities. The analysis considers the impact of the Rule on the allocation of resources between generations, abstracting from other effects of the Rule on the design of intergenerational transfers. It is now well understood (Friedman, 1988; Kaplow, 1995) that gratuitous transfers by rational actors fall below levels that maximize dynastic welfare. The problem arises because even altruistically motivated individuals fail to incorporate fully the welfare of recipients in making gratuitous transfers, and therefore transfer too little. As a result, there is scope for policies such as tax subsidies to encourage giving and thereby improve aggregate welfare (Farhi and Werning, 2010; Kopczuk, 2013).

The insight of Friedman (1988) and Kaplow (1995) that gratuitous transfers are insufficient to maximize dynastic welfare even when altruistically motivated carries particularly strong implications for the Rule against Perpetuities. An altruistically motivated transferor will typically consume too much; but the same individual, in choosing how to allocate transfers between members of subsequent generations, may well be inclined to do so in a way that maximizes their aggregate welfare and thereby maximizes his or her own. In that sense, someone who transfers to several succeeding generations will often have incentives to choose allocations among them in a manner akin to that of a social planner. The Rule against Perpetuities limits the range of available transfer options and thereby impedes the ability of transferors to impose binding restrictions on subsequent generations. As a result, the inherent selfishness of any receiving generation is given greater scope, with the predictable outcome that more distant generations are shortchanged. This relative impoverishment of distant generations generally implies that the Rule against Perpetuities reduces aggregate dynastic welfare, and indeed there are circumstances in which the Rule produces Pareto-inferior outcomes with lower utility in every generation.

\(^2\) See, for example, Gallanis (2000).
There are many economic settings in which rational choice supports efficient outcomes, whereas choice that is encumbered by legal restrictions does not. What complicates matters in the context of the Rule against Perpetuities is that rational intergenerational choice does not maximize dynastic welfare even in the absence of legal restrictions. As a result, there exist somewhat exceptional cases in which the Rule’s limitation on dispositive options can improve dynastic welfare. These cases arise when intergenerational preferences of the first generation differ sufficiently from those of the dynasty, or when in the absence of the Rule distant generations are so starved of resources that earlier generations feel compelled to bequeath much larger than desired sums in the expectation that some will trickle down to distant generations. Section 3 of the paper offers examples of welfare-improving application of the Rule, though section 4 identifies broad classes of rational preferences for which the Rule reduces dynastic welfare.

Some intergenerational transfers are surely motivated by forces other than rational and well-informed desires to advance the welfares of future generations, and it is entirely possible that in such circumstances restrictions on choice can enhance dynastic welfare. Indeed, some of the arguments commonly marshaled in support of the Rule against Perpetuities rely to one degree or another on suboptimal behavior by those who would prefer to make transfers that are proscribed by the Rule. If transfers to distant generations reflect mistaken calculation, inattention to the true welfare of future generations, or failure to anticipate changing economic or social circumstances then they typically will not constitute dynastically optimal plans. Since the Rule against Perpetuities reduces dynastic welfare in a broad class of cases with rational behavior, it follows that the normative case favoring the Rule relies on either the absence of rationality or else a social desire to maximize something other than dynastic welfare.

Section 2 of the paper compares intergenerational transfer incentives created by the Rule against Perpetuities with incentives in settings without the Rule. Section 3 analyzes examples that illustrate the effect of the Rule against Perpetuities on consumption and welfare, drawing attention to cases in which the Rule reduces the welfare of every generation, and those in which it actually improves dynastic welfare. Section 4 identifies restrictions on rational utility functions that are sufficient to guarantee that imposition of the Rule against Perpetuities reduces
dynastic welfare. Section 5 considers the implications of this analysis in the context of the lively debate over the desirability of the Rule against Perpetuities, and section 6 is the conclusion.

2. **Rational Gratuitous Behavior**

This section considers the conditions that characterize transfers to subsequent generations by rational actors who seek to enhance their own utilities. The point of the analysis is to compare outcomes with and without restrictions imposed by the Rule against Perpetuities.

It is useful to consider four successive generations of representative individuals, the distance between them of sufficient length that transfers attempting to skip a generation would run afoul of the Rule against Perpetuities. Individuals have the following utility functions:

\begin{align}
(1a) & \quad u^1(c_1, u^2, u^3, u^4) \\
(1b) & \quad u^2(c_2, u^3, u^4) \\
(1c) & \quad u^3(c_3, u^4) \\
(1d) & \quad u^4(c_4)
\end{align}

in which \( u^i \) is the utility level of generation \( i \), and \( c_i \) is consumption by generation \( i \). In the formulation in (1a)-(1d), the utilities of generations 2-4 enter the utility function of generation 1, the utilities of generations 3-4 enter the utility function of generation 2, and the utility of generation 4 enters the utility function of generation 3; only generation 4 is entirely selfish, in that it cares only about its own consumption.\(^3\) Generations are assumed to be altruistic and not envious, in that greater utility of subsequent generations contributes positively to the welfare of any current generation.

\(^3\) Utilities take the triangular form described in (1a)-(1d) in order to afford relatively straightforward characterization of behavior without concerns over multiple equilibria. A more general framework with more than four generations, each with utilities that depend on those of three subsequent generations, is likely to produce results that are
Consumption levels $c_1,\ldots,c_4$ are defined in present value terms, so an economy endowed with a fixed level of resources $y$ in the first period has a budget constraint of the form:

$$c_1 + c_2 + c_3 + c_4 \leq y.$$  

This specification omits explicit consideration of resources other than those that start with generation 1. Generations 2-4 may well have access to other resources, but the utility functions in (1b)-(1d) reflect that portion of welfare produced by consuming out of resources that originate with generation 1.

2.1 Dynastic welfare maximization

It is useful to start by considering the intergenerational allocation of consumption that maximizes welfare in this setting. Defining a Pareian dynastic welfare function,

$$\psi(u^1,u^2,u^3,u^4),$$

it follows that maximizing (3) over the choice of $c_i, \forall i=1,\ldots,4$, subject to (2), with utilities defined as in (1a)-(1d), and assuming interior solutions $(c_i > 0, \forall i)$, entails:

$$\frac{\partial \psi}{\partial u^i} \frac{\partial u^i}{\partial c_1} = \lambda$$

$$\frac{\partial u^2}{\partial c_2} \left[ \frac{\partial \psi}{\partial u^2} + \frac{\partial \psi}{\partial u^1} \frac{\partial u^1}{\partial u^2} \right] = \lambda$$

$$\frac{\partial u^3}{\partial c_3} \left[ \frac{\partial \psi}{\partial u^3} + \frac{\partial \psi}{\partial u^1} \frac{\partial u^1}{\partial u^3} + \frac{\partial \psi}{\partial u^2} \frac{\partial u^2}{\partial u^3} \right] = \lambda$$

$$\frac{\partial u^4}{\partial c_4} \left[ \frac{\partial \psi}{\partial u^4} + \frac{\partial \psi}{\partial u^1} \frac{\partial u^1}{\partial u^4} + \frac{\partial \psi}{\partial u^2} \frac{\partial u^2}{\partial u^4} + \frac{\partial \psi}{\partial u^3} \frac{\partial u^3}{\partial u^4} \right] = \lambda,$$

qualitatively similar to those analyzed in this paper, particularly in cases with significant intergenerational discounting.
in which $\lambda$ is the lagrange multiplier associated with the budget constraint (2). Denoting the welfare weight of generation $i$ relative to that of generation 1 by $\psi_i \equiv \frac{\partial \psi}{\partial \psi/\partial u^i}$, equations (4a) and (4b) together imply:

(5a) \[
\frac{\partial u^1/\partial c_1}{\partial u^2/\partial c_2} = \frac{\partial u^1}{\partial u^2} + \psi_2.
\]

Similarly, (4b) and (4c) together imply:

(5b) \[
\frac{\partial u^2/\partial c_2}{\partial u^3/\partial c_3} = \frac{\partial u^2}{\partial u^3} + \frac{\partial u^1}{\partial u^2} + \psi_3,
\]

and (4c) and (4d) together imply:

(5c) \[
\frac{\partial u^3/\partial c_3}{\partial u^4/\partial c_4} = \frac{\partial u^3}{\partial u^4} + \frac{\partial u^1}{\partial u^3} + \frac{\partial u^2}{\partial u^3} \left( \frac{\partial u^1}{\partial u^2} + \frac{\partial u^2}{\partial u^2} \right);
\]

These first-order conditions reflect the value of consumption by differing generations. For example, consumption of generation 2 produces utility for generation 2 and indirectly for generation 1; whereas consumption by generation 4 produces utility for generation 4 and for all of generations 1, 2 and 3. As a result, the dynastic planner prefers consumption by later generations to consumption by earlier generations, all other considerations equal. This is reflected for example in (5a), which has two terms on the right side: generation 1’s valuation of generation 2’s utility and the relative marginal weights of generation 2 and generation 1 in the dynastic welfare function. If generations 1 and 2 are equally weighted, so that $\psi_2 = 1$, then the right side of (5a) exceeds unity, implying that $\frac{\partial u^1}{\partial c_1} > \frac{\partial u^2}{\partial c_2}$ and generation 2 has a lower marginal utility of consumption than generation 1.

2.2. Individual choice without the Rule against Perpetuities
How well does uncoordinated individual behavior correspond to the optimality conditions expressed in (5a)-(5c)? It is instructive to consider the case in which the first generation commands all of the resources and is able to allocate them to maximize its own utility. In the absence of the Rule against Perpetuities, the first generation is able to transfer resources directly to distant generations 3 and 4, whereas the application of the Rule would ordinarily prevent such transfers.

An immediate potential complication is that the first generation cannot compel consumption on the part of intermediate generations 2 and 3. For example, generation 2 might save some of the resources it receives from generation 1, choosing to pass them along to generation 3 or 4. Since generations 3 and 4 receive their own transfers from generation 1, generation 2 will decide to make its own transfer if the value to generation 2 of improving the utility of one or more subsequent generations exceeds the cost in terms of foregone consumption by generation 2.

As it happens, this complication need not affect the calculation of generation 1, since in the absence of the Rule against Perpetuities it is able to allocate resources directly to generations 3 and 4. As a result, generation 1 can decide how much to give each subsequent generation, anticipating their behavior, and stifling their desire to transfer to subsequent generations by providing sufficient resources to those generations. Transfers that generation 2 might want to make to generation 3 would generally be welcomed by generation 1, which has an interest in the utilities of both generations, and indeed would be undertaken directly by generation 1. Consequently, for analytic purposes it is convenient to proceed as though the potential constraints of desired saving by generations 2-4 do not bind, that each subsequent generation consumes everything it receives from generation 1, as a result of which generation 1 can be treated as selecting the pattern of subsequent generational consumption.

A rational generation 1 member seeking to maximize (1a) over the choice of consumption in each period subject to (2), and again assuming interior solutions, sets:

\[
(6a) \quad \frac{\partial u^1}{\partial c_1} = \mu
\]
\[
\frac{\partial u^1}{\partial u^2} \frac{\partial u^2}{\partial c_2} = \mu \tag{6b}
\]

\[
\frac{\partial u^3}{\partial c_3} \left( \frac{\partial u^1}{\partial u^3} + \frac{\partial u^1}{\partial u^2} \frac{\partial u^2}{\partial u^3} \right) = \mu \tag{6c}
\]

\[
\frac{\partial u^4}{\partial c_4} \left[ \frac{\partial u^1}{\partial u^4} + \frac{\partial u^3}{\partial u^4} \left( \frac{\partial u^1}{\partial u^3} + \frac{\partial u^2}{\partial u^3} \frac{\partial u^1}{\partial u^2} \right) + \frac{\partial u^2}{\partial u^4} \frac{\partial u^1}{\partial u^2} \right] = \mu, \tag{6d}
\]

in which \( \mu \) is the lagrange multiplier associated with the budget constraint (2). These first-order conditions imply:

\[
\frac{\partial u^1}{\partial c_1} = \frac{\partial u^1}{\partial u^2} \tag{7a}
\]

\[
\frac{\partial u^2}{\partial c_2} = \frac{\partial u^2}{\partial u^3} + \frac{\partial u^1}{\partial u^3} \tag{7b}
\]

\[
\frac{\partial u^3}{\partial c_3} = \frac{\partial u^3}{\partial u^4} + \frac{\partial u^1}{\partial u^4} + \frac{\partial u^1}{\partial u^2} \frac{\partial u^2}{\partial u^4} \tag{7c}
\]

A comparison of equation (7a) with its welfare-maximizing counterpart equation (5a) reveals that generation 1’s marginal utility of consumption is greater, relative to the marginal utility of consumption of generation 2, for the allocation that maximizes welfare than for the allocation chosen by generation 1. This generally implies that dynastic welfare is maximized by offering generation 1 less consumption that it would choose if it had command of the dynasty’s resources.\(^4\) This is the point made by Friedman (1988) and Kaplow (1995): that despite the concern for subsequent generations evidenced in the utility function (1a), generation 1 in making

\(^4\) This inference from comparison of the marginal utilities expressed in (5a) and (7a) relies on the notion that a lower marginal utility of consumption corresponds to higher consumption levels. Since utilities are functions of utilities of future generations in addition to current consumption levels, it does not necessarily follow that it is possible to map from differences in marginal utilities of consumption to differences in consumption levels. The text uses a shorthand to describe the difference in marginal utilities as consumption by generation 1 in excess of the dynastic welfare maximizing level.
its allocation decisions fails to incorporate fully the effects of its actions on the welfares of
others. Generation 1 shares resources with subsequent generations in order to help itself, yet this
motivation is insufficiently other-regarding to maximize dynastic welfare; that is why the
\( \psi_2 \) term, which appears on the right side of (5a), and captures the dynastic valuation of
generation 2’s utility, is missing from the right side of (7a) that characterizes the allocation
chosen by generation 1. Generation 1 cares about the welfare of generation 2, and that concern
is incorporated in its decisions; but generation 1 is not influenced by generation 2’s welfare
independent of generation 1’s valuation – whereas the dynasty is – and as a result, generation 1
consumes too much.

In addition to choosing its own consumption, generation 1 allocates consumption
between generations 2 and 3, doing so to maximize generation 1’s utility. As noted, the
allocations chosen by generation 1 will generally entail too much consumption by generation 1,
and too little consumption by subsequent generations, relative to levels that would maximize
dynastic welfare. Conditional on these low levels of generation 2 and 3 consumption, the
allocation chosen by generation 1 may closely approximate that selected by a planner seeking to
maximize dynastic welfare. The right side of equation (5b), characterizing the optimal level of
\[ \frac{\partial u^2}{\partial c_2} \bigg/ \frac{\partial u^3}{\partial c_3} \]
the difference between the terms \[ \frac{\partial u^1}{\partial u^2} \] and \[ \frac{\partial u^1}{\partial u^3 + \psi_3} \]. The first of these terms is generation
1’s relative valuation of generation 3 and generation 2 utilities; the second reflects differences in
the dynastic valuation of generation 3 and generation 2 utilities. If generation 1 weighs these
marginal utilities equally, so that \[ \frac{\partial u^1}{\partial u^3} = \frac{\partial u^1}{\partial u^2} \], and the dynastic planner likewise weighs them
equally, so that \[ \psi_3 = \psi_2 \], then the right sides of (5b) and (7b) are equal. It would then follow that
generation 1’s choice of the marginal tradeoff between consumption of generations 2 and 3
equals the marginal tradeoff that would be chosen by a planner who maximizes dynastic welfare.
A similar logic applies to the ratios $\frac{\partial u^3/\partial c_3}{\partial u^4/\partial c_4}$ chosen by generation 1 and by the dynastic planner. In comparing the right sides of (5c) and (7c), it is clear that if $\frac{\partial u^1}{\partial u^4} = \frac{\partial u^1}{\partial u^3}$, $\frac{\partial u^2}{\partial u^4} = \frac{\partial u^2}{\partial u^3}$, and $\psi_4 = \psi_3$, then $\frac{\partial u^3/\partial c_3}{\partial u^4/\partial c_4} = \frac{\partial u^3}{\partial u^4} + 1$, both in the allocation chosen by the dynastic planner and in the allocation chosen by generation 1. These conditions require that members of generations 1 and 2, as well as the dynastic planner, value the marginal utilities of generations 3 and 4 equally.

Future consumption choices made by generation 1 resemble those that would be made by a dynastic planner because, given the structure of the problem, both are motivated by desire to maximize the utilities of future generations subject to limits imposed by resource constraints. They differ in two respects: generation 1 allocates itself greater consumption than would the dynastic planner, and generation 1 chooses an allocation among members of generations 2-4 based on generation 1’s utility weights rather than the dynastic planner’s utility weights. While there may be little a priori reason to think that generation 1’s relative valuation of the welfares of different subsequent generations systematically deviates from the dynastic planner’s relative valuation, it is important to note that these valuations correspond to preferences with different origins, and therefore have the potential to differ significantly.

2.3. Individual choice with the Rule against Perpetuities

What if transfers are subject to the Rule against Perpetuities? Given the model’s specification, it follows that the Rule permits transfers only to members of the immediately succeeding generation. In the case of nonzero transfers made by generation 3 to generation 4, the Rule is not implicated, and these transfers must satisfy:

$$ (8a) \quad \frac{\partial u^3}{\partial c_3} = \frac{\partial u^4}{\partial c_4} \frac{\partial u^3}{\partial u^4}. $$

Transfers made by generation 2 to generation 3 depend in part on the extent to which generation 2 anticipates that marginal funds will be subsequently transferred by generation 3 to generation 4, as generation 2 has an interest in generation 4’s welfare. Denoting by $\gamma_3$ the
fraction of marginal bequests received by generation 3 that are subsequently transferred to
generation 4, generation 2’s optimization over nonzero transfers entails setting:

\[
\frac{\partial u^3}{\partial c_2} = \frac{\partial u^2}{\partial u'} \frac{\partial u^3}{\partial c_3} (1 - \gamma_3) + \gamma_3 \frac{\partial u^2}{\partial c_4} \left( \frac{\partial u^3}{\partial u'} + \frac{\partial u^2}{\partial u'} \frac{\partial u^3}{\partial u'} \right) = \frac{\partial u^2}{\partial u'} \frac{\partial u^3}{\partial c_3} + \gamma_3 \frac{\partial u^2}{\partial c_4} \frac{\partial u^2}{\partial u'},
\]

in which the right side of (8b) incorporates (8a). Denoting by \( \gamma_2 \) the fraction of marginal receipts of generation 2 that are subsequently transferred to generation 3, nonzero transfers from generation 1 to generation 2 must satisfy:

\[
\frac{\partial u^1}{\partial c_1} = \frac{\partial u^2}{\partial u'} \frac{\partial u^1}{\partial c_2} (1 - \gamma_2) + \gamma_2 \frac{\partial u^3}{\partial c_3} \left( \frac{\partial u^3}{\partial u'} + \frac{\partial u^2}{\partial u'} \frac{\partial u^3}{\partial u'} \right) (1 - \gamma_3) + \gamma_2 \gamma_3 \frac{\partial u^4}{\partial c_4} \left[ \frac{\partial u^1}{\partial u'} + \frac{\partial u^1}{\partial u'} \frac{\partial u^3}{\partial u'} + \frac{\partial u^1}{\partial u'} \left( \frac{\partial u^2}{\partial u'} + \frac{\partial u^2}{\partial u'} \frac{\partial u^2}{\partial u'} \right) \right] .
\]

Equations (8a)-(8c) together imply:

\[
\begin{align*}
\frac{\partial u^1}{\partial c_1} &= \frac{\partial u^1}{\partial u'} + \gamma_2 \frac{\partial u^2}{\partial u'} \frac{\partial u^1}{\partial c_2} + \gamma_2 \frac{\partial u^3}{\partial c_3} \left( \frac{\partial u^3}{\partial u'} + \frac{\partial u^2}{\partial u'} \frac{\partial u^3}{\partial u'} \right) + \gamma_2 \gamma_3 \frac{\partial u^4}{\partial c_4} \left[ \frac{\partial u^1}{\partial u'} + \frac{\partial u^1}{\partial u'} \frac{\partial u^3}{\partial u'} + \frac{\partial u^1}{\partial u'} \left( \frac{\partial u^2}{\partial u'} + \frac{\partial u^2}{\partial u'} \frac{\partial u^2}{\partial u'} \right) \right] , \\
\frac{\partial u^2}{\partial c_2} &= \frac{\partial u^2}{\partial u'} + \gamma_3 \frac{\partial u^2}{\partial u'} \frac{\partial u^4}{\partial c_4} , \\
\frac{\partial u^3}{\partial c_3} &= \frac{\partial u^4}{\partial c_4} .
\end{align*}
\]

Equations (9a)-(9c) describe an allocation that differs significantly from that chosen by
generation 1 in the absence of the Rule against Perpetuities. The most obvious difference lies in
comparing (9c) to (7c): the right sides of these two first-order conditions characterizing
\( \frac{\partial u^3}{\partial c_2} \) differ by the second term on the right side of (7c), which is positive, and indeed in an
important class of cases equal to one. Consequently, the ratio of the marginal utility of
consumption by generation 3 to the marginal utility of consumption by generation 4 exceeds the
cost that the Rule against Perpetuities is responsible for a rather threadbare existence for generation 4, which is entitled to
consumption only to the extent that it generates utility that benefits generation 3. By contrast, the allocation chosen by generation 1 in the absence of the Rule against Perpetuities, and
described in (7c), will generally offer considerably more to generation 4 relative to generation 3, reflecting that generation 4’s utility enhances its own well-being and that of generations 2 and 3, all of whose welfares benefit generation 1. Since the dynastic welfare maximizing outcome

described by (5c) also entails a significantly higher value of $\frac{\partial u^3}{\partial c_4}$ than that implied by (9c), it
is likely that, on this margin, the Rule reduces dynastic welfare.

A comparison of the values of $\frac{\partial u^2}{\partial c_2}$ with and without the Rule against Perpetuities depends in part on the value of $\gamma_3$, a variable that captures the proclivity of generation 3 to
transfer marginal receipts to generation 4. Given the scant regard for generation 4’s welfare expressed by (9c), $\gamma_3$ is it is unlikely to take a large value, and almost surely lies below 0.5, since otherwise generation 3 would allocate more marginal resources to generation 4 than to itself. If $\gamma_3 = 0$ then the right side of (9b) omits a positive term that appears in the right side of (7b).

Consequently, if generation 1’s relative valuation of the utilities of generations 2 and 3,

$$\frac{\partial u^1}{\partial u^3}$$

is comparable to the ratio of generation 2 and 3’s valuation of the utility of generation

$$\frac{\partial u^2}{\partial u^4}$$,

then if $\frac{\partial u^3}{\partial u^4}$ is unchanging it follows that the small value of $\gamma_3$ implies that the right side of (9b) is smaller than the right side of (7b). As a result, the Rule against Perpetuities reduces consumption by generation 3 relative to consumption by generation 2.

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5 It is reasonable to assume that $\frac{\partial u^3}{\partial u^4}$ takes a positive value considerably smaller than one, since it is unlikely that generation 3 would value generation 4’s utility comparably to its own.
The comparison of the values of $\frac{\partial u_1}{\partial c_1} / \frac{\partial u_2}{\partial c_2}$ with and without the Rule against Perpetuities depends additionally on the value of $\gamma_2$. A comparison of (9a) with (7a) indicates that the implied ratio is greater with the Rule against Perpetuities than without, this result reflecting that generation 1 must work through generation 2 in order to transfer resources to generation 3 and ultimately to generation 4. Since generation 2 transfers only a fraction of its resources to generation 3, generation 1 must provide sufficient transfers to generation 2 so that some will go to generation 3 and ultimately seep through to generation 4.

As a general matter, the comparison of (9a) and (7a) suggests that imposition of the Rule against Perpetuities increases the consumption of generation 2. It does not, however, follow that imposition of the Rule enhances the utility of generation 2, because resources are misallocated at every generation. Generation 2 faces a version of the same problem that confronts generation 1, in that it cannot transfer directly to generation 4, but must instead go through generation 3. As a result, there are scenarios in which imposition of the Rule against Perpetuities reduces the welfare of every generation. Alternatively, if generation 1 has preferences over the welfares of subsequent generations that are seriously misaligned with dynastic preferences, then imposition of the Rule could improve the welfare of each subsequent generation and the dynasty as a whole.

3. Illustrative Examples

The impact of the Rule against Perpetuities can be illustrated by considering outcomes for generations that have utility functions that are simple sums of the discounted welfares of subsequent generations plus the natural log of current consumption. Generation 1’s utility is given by: $\ln c_1 + \alpha \beta u^2 + \alpha^2 \gamma u^3 + \alpha^3 \delta u^4$, in which $0 \leq \alpha, \beta, \gamma, \delta \leq 1$, and $\alpha$ can be interpreted as a discount factor; generation 2’s utility is given by $\ln c_2 + \alpha u^3 + \alpha^2 u^4$, generation 3’s by $\ln c_3 + \alpha u^4$, and generation 4’s by $\ln c_4$. Furthermore, assume that dynastic welfare is given by the discounted sum of utilities: $u_1 + \alpha u^2 + \alpha^2 u^3 + \alpha^3 u^4$. In this formulation, generation 1’s valuation
of the utilities of future generations 2-4 differs from the valuations of others, including the
dynastic planner, to the extent that the values of $\beta$, $\gamma$ and $\delta$ differ from unity.

In the absence of a Rule against Perpetuities, generation 1 chooses each generation’s
consumption to maximize: $\ln c_1 + \alpha \beta \ln c_2 + \alpha^2 (\beta + \gamma) \ln c_3 + \alpha^3 (2 + \gamma + \delta) \ln c_4$. Maximizing
this function over the choice of consumption levels, subject to $c_1 + c_2 + c_3 + c_4 \leq \gamma$, implies that:

\[
(10) \quad c_1 = \frac{1}{\alpha \beta} c_2 = \frac{1}{\alpha^2 (\beta + \gamma)} c_3 = \frac{1}{\alpha^3 (2 \beta + \gamma + \delta)} c_4,
\]

and as a result, $c_1 = y\sqrt{1 + \alpha \beta + \alpha^2 (\beta + \gamma) + \alpha^3 (2 \beta + \gamma + \delta)}$. Furthermore, in this example,
dynastic welfare equals $\ln c_1 + \alpha (1 + \beta) \ln c_2 + \alpha^2 (2 + \beta + \gamma) \ln c_3 + \alpha^3 (4 + 2 \beta + \gamma + \delta) \ln c_4$, hence
is maximized by setting:

\[
(11) \quad c_1 = \frac{1}{\alpha (1 + \beta)} c_2 = \frac{1}{\alpha^2 (2 + \beta + \gamma)} c_3 = \frac{1}{\alpha^3 (4 + 2 \beta + \gamma + \delta)} c_4.
\]

The allocation that maximizes dynastic welfare entails a distribution of consumption
among generations 2-4, expressed in (11), that is similar (in proportion, though not in scale) to
the distribution (10) chosen by generation 1 in the absence of the Rule against Perpetuities. The
two distributions are exactly proportional if $\beta = \gamma = \delta = 1$, in which case generation 1 prefers
that consumption be allocated among generations 2-4 in a way that maximizes their aggregate
contribution to dynastic welfare, but also prefers that aggregate consumption of generations 2-4
lies below the level that maximizes dynastic welfare, thereby permitting greater consumption by
generation 1.

The Rule against Perpetuities, in prohibiting direct transfers to distant beneficiaries,
affords greater discretion to intermediate generations, thereby changing outcomes. Generation 3
maximizes $\ln c_3 + \alpha \ln c_4$, hence allocates its resources so that $c_4 = \alpha c_3$. Generation 2 chooses
$c_2$ to maximize $\ln c_2 + \alpha \ln c_3 + 2 \alpha^2 \ln c_4$, from which it follows, given $c_4 = \alpha c_3$, that generation 2
chooses its consumption level so that $c_3 = \frac{3\alpha}{(1+\alpha)} c_2$. Imposing these conditions on generation 1’s choice of $c_1$, it follows from generation 1’s first order condition that:

$$c_1 = \frac{(1+3\alpha)}{\theta} c_2 = \frac{(1+\alpha)(1+3\alpha)}{3\alpha \theta} c_3 = \frac{(1+\alpha)(1+3\alpha)}{3\alpha^2 \theta} c_4,$$

in which $\theta = \left[ \alpha \beta + \alpha^2 (\beta + \gamma) + \alpha^3 (2\beta + \gamma + \delta) \right]$. Imposing the budget constraint (2) together with (12) implies that $c_1 = \frac{y}{(1+\alpha \beta + \alpha^2 (\beta + \gamma) + \alpha^3 (2\beta + \gamma + \delta))}$, which is identical to the first-generation consumption level in the absence of the Rule against Perpetuities. From the standpoint of dynastic welfare, there is too much first-generation consumption in the absence of the Rule against Perpetuities, but in this example resources are otherwise allocated properly if $\beta = \gamma = \delta = 1$. Introduction of the Rule against Perpetuities does not address the problem of excessive first-period consumption, as this consumption level does not change, but the Rule entails distorting the pattern of consumption by generations 2-4.

It is useful first to consider the case of $\beta = \gamma = \delta = 1$, in which generation 1’s valuations are consistent with those of future generations and the dynastic planner. Table 1 presents numerical calculations of implied consumption and welfare for the case in which future utilities are not discounted, so that $\alpha = 1$. The first row of Panel A presents consumption levels of each generation when transfers are limited by the Rule against Perpetuities. Generation 1 chooses to consume just one-eighth of the available resources, bequeathing the remainder in the (justified) expectation that consumption by subsequent generations will produce more utility for generation 1 than would its own greater consumption. Generation 2 is less unselfish, consuming one-quarter of what it receives, and leaving the rest to generation 3, which shares evenly with generation 4.

Generation 1 would allocate resources differently if not constrained by the Rule against Perpetuities, its preferred allocation appearing in the second row of Panel A. Generation 1’s own consumption is again just one-eighth of available resources, but generation 2’s would also be limited to one-eighth of the total, with generation 3 consuming one-quarter and generation 4 – whose consumption benefits generation 1 in part by augmenting the utilities of generations 2 and
3 – consuming one-half. It is instructive to compare the unconstrained consumption choices of generation 1 with those corresponding to dynastic welfare maximization, presented in the third row of Panel A. The consumption pattern that maximizes dynastic welfare entails consumption doubling in every succeeding generation, reflecting the greater contribution of later generations to dynastic welfare through their effect on the utilities of earlier generations. This pattern is the product of welfare functions in which individual utilities are functions of the utilities of descendants but not the utilities of ancestors. The pattern in which consumption of generation 3 is double that of generation 2, and consumption of generation 4 is double that of generation 3, is common to dynastic welfare maximization (row 3) and generation 1’s bequests in the absence of the Rule against Perpetuities (row 2). Generation 1 attaches the same relative valuation to utilities of distant generations as a dynastic planner would – which is why generation 1, if able to impose binding bequest constraints, would mimic the planner’s allocation among generations 2-4. The only difference is that generation 1 allocates itself more consumption than would a dynastic planner.

Panel B of Table 1 presents utility levels implied by the consumption allocations reported in Panel A. All four generations have higher utility in the absence of the Rule against Perpetuities than when choices are constrained by the Rule. This outcome is obvious in the case of generation 1, and is unsurprising in the cases of generations 3 and 4, which are spared the ravages of generation 2’s consumption urges when generation 1 is unconstrained by the Rule. The striking aspect is the outcome for generation 2, which obtains greater utility \(4 \ln(y) - 4.852\) in the absence of the Rule against Perpetuities than it does with the Rule \(4 \ln(y) - 4.863\). Since in this example generation 1’s consumption is the same with or without the Rule against Perpetuities, it might seem odd that generation 2 does better in the absence of the Rule, when generation 1 chooses its consumption, than it does with the Rule, in which case generation 2 can decide how much to consume of the \(\frac{7}{8} y\) legacy that it receives from generation 1. The problem, familiar to every parent, is the inability to control the next generation. In the absence of the Rule, generation 3 will divide its resources evenly between itself and generation 4, whereas generation 2 would prefer generation 3 to consume one-third and leave the remaining two-thirds to generation 4. Anticipating an even division, generation 2 increases its own
consumption, but even with this adjustment its utility falls short of the level it would obtain in the absence of the Rule against Perpetuities, when generation 1 would (comparatively) shortchange generation 2’s consumption but more than make up for it with generation 4’s consumption. Since the utility of every generation declines with the imposition of the Rule against Perpetuities, dynastic welfare also declines.

Table 2 presents consumption and welfare levels in the same model when $\alpha = 0.5$, so that future utilities are discounted by one-half per generation. With greater discounting of future utilities consumption profiles are more strongly oriented toward earlier generations than was the case with $\alpha = 1$. When constrained by the Rule against Perpetuities, generation 1 consumes 40 percent of available resources; generations 2 and 3 follow by each consuming 24 percent of total resources, and generation 4 consumes just 12 percent. In the absence of the Rule against Perpetuities generation 1 again consumes 40 percent of available resources, and generations 2-4 consume 20 percent each. The allocation that maximizes dynastic welfare has each generation consuming one-quarter of the economy’s endowment, a pattern of equality among generations 2-4 similar that chosen by generation 1 in the absence of the Rule against Perpetuities. Indeed, the only difference between the allocation that maximizes dynastic welfare and that chosen by generation 1 in the absence of the Rule is that in the latter case generation 1 increases its own consumption to a level double that of any subsequent generation.

Dynastic welfare increases with removal of the Rule against Perpetuities, though not every generation fares better: generation 2 has greater utility with the Rule than without. The welfare effect of the Rule against Perpetuities reflects that higher discount rates increase the importance of consumption by earlier generations. As a result of greater discounting, the higher consumption level of generation 4 (20 percent of total resources in the absence of the Rule against Perpetuities compared to 12 percent with the Rule) is insufficient to compensate generation 2 for its own reduced consumption (20 percent of total resources in the absence of the Rule against Perpetuities compared to 24 percent with the Rule). And as is also characteristic of the $\alpha = 1$ case presented in Table 1, every generation other than the first has higher utility in the

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6 The consumption level reported in the first row of Panel A implies that generation 2 consumes one-quarter of the legacy it receives from generation 1. It is straightforward to show that if generation 3 were constrained to leave two-thirds of its resources to generation 4, then generation 2 would choose to consume just one-sixth of what it receives from generation 1.
program that maximizes dynastic welfare than in the absence of the Rule against Perpetuities. In the $\alpha = 0.5$ case generation 1 also fares better without the Rule than it does when consumption is chosen to maximize dynastic welfare, again reflecting the greater weight on the benefits of its own consumption compared to the utilities of later generations.

It is also useful to consider the welfare properties of the Rule against Perpetuities in cases in which generation 1’s preferences differ from those of other generations and the preferences of the dynastic planner, so that values of $\beta, \gamma$ and $\delta$ differ from one. Table 3 describes outcomes with differing parameter values, noting whether the Rule against Perpetuities or its absence yields greater dynastic welfare. The first three rows consider cases in which $\beta = \gamma = \delta = 0.2$, so that generation 1 systematically underweights the utilities of future generations. In the cases of all three of the discount factors reported in the table ($\alpha = 1, \alpha = 0.5, \alpha = 0.1$), dynastic welfare is greater in the absence of the Rule against Perpetuities. The same is true in the exercises, reported in rows 4-6 of Table 3, in which generation 1 fully values generation 2’s utility ($\beta = 1$), but does not directly value generation 3 and 4’s utility ($\gamma = \delta = 0$). It is noteworthy that, in the cases with $\gamma = \delta = 0$, generation 1 in the absence of the Rule against Perpetuities nonetheless devotes resources for generations 3 and 4, since their utility indirectly benefits generation 1 by augmenting generation 2’s utility.

Rows 7-9 of Table 3 present cases in which generation 1 systematically undervalues generation 2’s utility, with $\beta = 0.2$ and $\gamma = \delta = 1$. Dynastic welfare is higher in the absence of the Rule against Perpetuities if $\alpha = 1$ or $\alpha = 0.5$, but as the discount factor declines so too does the relative cost of the Rule, and if $\alpha = 0.1$ then dynastic welfare is greater with the Rule against Perpetuities than without. Recall that consumption by generation 1 is the same in these examples with or without the Rule against Perpetuities, so the welfare differences stem from consumption patterns among generations 2-4. With the Rule against Perpetuities generation 2 consumes too much, and generation 3 also consumes too much of the bequest it receives from generation 2. In the absence of the Rule against Perpetuities, generation 2 consumes too little, because generation 1, which undervalues generation 2’s consumption relative to generations 3 and 4, allocates too little to generation 2. Smaller values of $\alpha$ reduce the relative importance of the misallocation between generations 3 and 4 produced by the Rule against Perpetuities, and dampen the extent to
which generation 2 over-consumes relative to the dynastic optimum, ultimately making the Rule against Perpetuities welfare-enhancing compared to its absence.

Rows 10-12 of Table 3 describe exercises that are similar to those displayed in rows 7-9, with the difference that generation 1 now even more severely undervalues generation 2’s utility \((\beta = 0.1)\) while remaining attentive to the utilities of generations 3 and 4 \((\gamma = \delta = 1)\). Dynastic welfare is again greater in the absence of the Rule against Perpetuities if \(\alpha = 1\), but if \(\alpha = 0.5\) or \(\alpha = 0.1\) then dynastic welfare is greater with the Rule against Perpetuities than without. This again reflects that generation 2 consumes too little in the absence of the Rule against Perpetuities, a problem that intensifies at lower values of \(\beta\), and is made relatively worse with low discount factors.

4. **Functional Restrictions and Welfare Effects of the Rule against Perpetuities**

This section identifies restrictions on utility functions that are sufficient to guarantee that dynastic welfare is greater in the absence of the Rule against Perpetuities.

It is useful to start by considering settings in which transferors have truly altruistic preferences over the relative welfares of future generations, weighting the welfares of others – relative to each other – in a way that a dynastic planner would. In the model’s specification, a generation’s utility is a function of its own consumption and the utilities of future generations, which are in turn functions of future consumption levels. Consequently, it is possible to express utility as a function of present and future consumption levels. Other-regarding preferences can be captured by expressing individual 1’s utility as being weakly separable in own consumption and future consumption:

\[
(13) \quad u^1[c_1, \phi(c_2, c_3, c_4)],
\]

in which the function \(\phi(c_2, c_3, c_4)\) is increasing in all of its arguments and \(u^1[\cdot]\) is quasi-concave, so that \(\frac{\partial u^1}{\partial c_1}/\frac{\partial c_1}{\phi}\) declines with \(\frac{c_1}{\phi}\) for a given utility level. The restriction imposed by (13) over
the formulation in (1a) is that in (13) the level of first period consumption does not affect rates at which generation 1 trades off the relative values of consumption by generations 2-4. Dynastic welfare is assumed to be capable of being expressed similarly:

\[
(14) \quad \psi \left[ u^1, \phi(c_2, c_3) \right].
\]

Given the utility function expressed in (13), individual 1 chooses transfers to maximize the joint welfare of generations 2-4. From this Proposition 1 immediately follows:

**PROPOSITION 1:** If individual utility and dynastic welfare take the forms expressed in (13) and (14), and if the Rule against Perpetuities improves dynastic welfare, then consumption in the first period must be lower under the Rule against Perpetuities than in the absence of the Rule.

Proof: The function \( \psi \left( u^1, \phi \right) \) is increasing in both of its arguments, so in order for dynastic welfare to be higher with the Rule against Perpetuities than without the Rule it must be the case that either \( u^1 \) or \( \phi \) is greater with the Rule. Since generation 1 chooses \( (c_1, c_2, c_3, c_4) \) to maximize its own utility in the absence of the Rule, \( u^1 \) cannot rise with the imposition of the Rule. Hence \( \phi \) must rise; but since generation 1 chooses \( (c_2, c_3, c_4) \) to maximize \( \phi \) conditional on available resources \( (y - c_1) \), \( \phi \) cannot rise with the imposition of the Rule unless \( (y - c_1) \) rises, which requires \( c_1 \) to decline.

Proposition 1 reflects that if generation 1 chooses consumption among generations 2-4 in the same way as would a dynastic planner, then application of the Rule against Perpetuities can improve welfare only through its effect on the allocation between generation 1 and all others. Since generation 1’s welfare cannot improve with the imposition of a restriction on its range of options, it follows that the welfare of generations 2-4 would have to rise in order for dynastic welfare to increase, and that will happen only with additional resources released by reduced consumption of generation 1. But from the standpoint of generation 1, imposition of the Rule against Perpetuities reduces the quality of resource allocation among subsequent generations, very possibly reducing its willingness to make transfers to generation 2. A competing
consideration is that this misallocation reduces the utilities of distant generations, possibly making it even more valuable to make marginal transfers that would increase their welfare. In this scenario generation 2 effectively holds generations 3 and 4 hostage to its own selfishness, thereby encouraging greater largesse on the part of generation 1. A similar process operates on generation 2, which can transfer to generation 4 only via generation 3.

If imposition of the Rule against Perpetuities increases dynastic welfare, then it must be the case that, upon imposition of the Rule, generation 1 chooses to reduce its own consumption in order to provide greater resources for subsequent generations, and continue doing so beyond the point at which subsequent generations receive so many additional resources that they are, in aggregate, better off than they were before the Rule was imposed. Since from generation 1’s standpoint the Rule causes a misallocation of resources among generations 2-4, it may require considerable extra bequests to restore the initial level of welfare; and due to the misallocation, marginal bequests may not generate much in the way of additional welfare. The unlikely-looking scenario in which the Rule improves aggregate welfare requires that marginal bequests from generation 1 not only augment the resources of generations 2-4 but also cause a reallocation of their relative consumption levels. This notion can be made more precise by considering an important class of cases in which such reallocations do not occur.

It is useful to rewrite the utility functions for generations 2 and 3, presented originally in (1b) and (1c), as functions of own and subsequent consumption:

(15b) \[ u^2(c_2, c_3, c_4) \]

(15c) \[ u^3(c_3, c_4). \]

Proposition 2 then identifies the impact of the Rule against perpetuities when utilities are homothetic functions and therefore the relative consumption of different generations does not change with income.
PROPOSITION 2: If individual utility and dynastic welfare take the forms expressed in (13) and (14), and if \( u^1\left[c_1, \phi(c_2, c_3, c_4)\right], \phi(c_2, c_3, c_4), u^2(c_2, c_3, c_4) \) and \( u^3(c_3, c_4) \) are all homothetic functions, then the Rule against Perpetuities cannot increase dynastic welfare.

PROOF: The first step in the proof is to verify that, with these homothetic preferences, relative consumption patterns among generations 2-4, in scenarios with and without the Rule against Perpetuities, are unaffected by generation 1’s consumption level. In the absence of the Rule against Perpetuities this follows immediately from generation 1’s utility maximization problem. If \( \phi(c_2, c_3, c_4) \) is a homothetic function, then without loss of generality it can be expressed as

\[
g \left( \frac{c_2}{z_1}, \frac{c_3}{z_1}, \frac{c_4}{z_1} \right)
\]

in which \( g \left( \frac{c_2}{z_1}, \frac{c_3}{z_1}, \frac{c_4}{z_1} \right) \) is a quasi-concave function and \( z_1 = y - c_1 \) is the bequest that generation 1 leaves to all subsequent generations. Generation 1 chooses \( c_2, c_3, c_4 \) to maximize the value of \( g \left( \frac{c_2}{z_1}, \frac{c_3}{z_1}, \frac{c_4}{z_1} \right) \), and these choices are clearly unaffected by the value of \( z_1 \).

Next consider outcomes with the Rule against Perpetuities. Generation 3’s utility can be expressed as

\[
z_2 g_3 \left( \frac{c_3}{z_2}, \frac{c_4}{z_2} \right),
\]

in which \( g_3 \left( \frac{c_3}{z_2}, \frac{c_4}{z_2} \right) \) is a quasi-concave function and \( z_2 \) is the bequest left by generation 2. It follows that generation 3’s utility-maximizing choices of \( \frac{c_3}{z_2} \) and \( \frac{c_4}{z_2} \) are independent of the level of \( z_2 \). Similarly, \( u^2(c_2, c_3, c_4) \) can be expressed as

\[
z_1 g_2 \left( \frac{c_2}{z_1}, \frac{c_3}{z_1}, \frac{c_4}{z_1} \right).
\]

Generation 2 chooses \( \frac{c_2}{z_1} \) to maximize \( g_2 \left( \frac{c_2}{z_1}, \frac{c_3}{z_1}, \frac{c_4}{z_1} \right) \), subject to the constraint that generation 3 selects \( \frac{c_3}{z_1 - c_2} \) and \( \frac{c_4}{z_1 - c_2} \) independent of the level of \( \frac{c_3}{z_1} \). This maximization determines \( \frac{c_2}{z_1} \)

and as a result, in the absence of the Rule against Perpetuities \( \frac{c_2}{z_1}, \frac{c_3}{z_1}, \frac{c_4}{z_1} \) are all independent of the level of \( z_1 \).

Since \( z_1 = y - c_1 \), it follows that generation 1’s utility can be expressed as

\[
u^1 \left[ c_1, (y - c_1) g \left( \frac{c_2}{z_1}, \frac{c_3}{z_1}, \frac{c_4}{z_1} \right) \right].
\]

Generation 1’s first-order condition for the welfare-maximizing level of \( c_1 \) is therefore:
Homotheticity of the \( u^1[\phi(c_2, c_3, c_4)] \) function implies that the left side of (16) is constant along a ray from the origin and hence at any utility level. Proposition 1 implies that, in order for welfare to rise with imposition of the Rule against Perpetuities, it must be the case that \( \phi \) increases and \( c_1 \) declines, which from the quasi-concavity of the \( u^1[\phi(c_2, c_3, c_4)] \) function it follows that the \( \frac{\partial u^1}{\partial \phi} \) ratio on the left side of (16) must rise. But since the Rule restricts the range of choices open to generation 1, \( g \left( \frac{c_2}{z_1}, \frac{c_3}{z_1}, \frac{c_4}{z_1} \right) \) cannot increase with the imposition of the Rule. Hence there is a contradiction: starting from the absence of the Rule, if imposition of the Rule increases dynastic welfare, it increases the left side of (16) and reduces the right side of (16), making it impossible for (16) to continue to hold. Consequently, the Rule against Perpetuities cannot increase dynastic welfare.

Proposition 2 reflects that, from the standpoint of generation 1, the Rule against Perpetuities reduces the quality of resource allocation among generations 2-4, effectively making it more expensive to obtain utility from future generations. Generation 1 responds to this greater cost by reducing the utility that it obtains from future generations, as a result of which dynastic welfare declines.

Proposition 2 applies only in cases of homothetic utility functions, leaving open the possibility that the Rule against Perpetuities could improve dynastic welfare with sufficiently non-uniform income effects. Specifically, in the absence of the Rule against Perpetuities there exist cases in which generations 2 and 3 consume many of the first resources bequeathed by generation 1, thereby leaving generation 4 to a threadbare existence. If marginal bequests by generation 1 would trigger a significant reallocation of consumption in favor of generation 4, then generation 1 might have an incentive further to reduce its own consumption, prompting an improvement in generation 4’s prospects and possibly a net improvement in dynastic welfare compared to the outcome in the absence of the Rule against Perpetuities. This scenario relies on generations 2 and 3 responding to marginal bequests from generation 1 by drastically changing their consumption patterns, which is unlikely often to be the case, but it is possible.
The analysis in this section suggests that there is a broad class of circumstances in which the Rule of Perpetuities reduces dynastic welfare. Importantly, this analysis depends on the assumption that generation 1 values the relative welfares of generations 2-4 in the same way that a dynastic planner would, albeit against a background in which generation 1 may attach significantly greater value to its own consumption. In the absence of the Rule against Perpetuities generation 1 has roughly dictatorial control over the disposition of its own assets. This need not be all bad from the standpoint of dynastic welfare, particularly if the alternative is control dispersed among subsequent generations that each consume more than the dynastically optimal level. But it is worth bearing in mind that generation 1 might have preferences over future consumption that deviate significantly from what would be preferred by future generation or a planner who seeks to maximize the welfare of the dynasty.

It is possible to identify the welfare properties of certain cases in which generation 1’s preferences deviate in certain ways from those of the dynastic planner. Proposition 3 concerns cases in which generation 1 systematically under- or over-values consumption by generations 2-4 to the same degree; Proposition 4 concerns cases in which generation 1 does not care about the utilities of generations 3 and 4, and generation 2 values the utilities of generations 3 and 4 to the same degree as does the dynastic planner. In both sets of cases introduction of the Rule against Perpetuities cannot increase dynastic welfare.

**PROPOSITION 3:** If generation 1’s utility is a homothetic function \( u^1(c_1, \phi(c_2, c_3, c_4)) \); the functions \( \phi(c_2, c_3, c_4) \), \( u^2(c_2, c_3, c_4) \) and \( u^3(c_3, c_4) \) are also homothetic; and dynastic welfare can be expressed as \( \psi[u^1, \phi(kc_2, kc_3, kc_4)] \); then the Rule against Perpetuities cannot increase dynastic welfare.

**PROOF:** The proof follows directly from the proofs of Propositions 1 and 2. Since \( \phi(c_2, c_3, c_4) \) is homothetic, then dynastic welfare in this instance it can be expressed as

\[
\psi \left[ u^1, (z,k) g \left( \frac{c_2}{z_1}, \frac{c_3}{z_1}, \frac{c_4}{z_1} \right) \right],
\]

and since \( k \) is a constant the proofs of Propositions 1 and 2 then apply directly.
Proposition 3 indicates that if generation 1’s valuation of generation 2-4 consumption differs by a scalar factor from that of the dynastic planner, and utility functions are homothetic and therefore do not exhibit unusual income effects, then the Rule against Perpetuities cannot improve welfare. This is quite consistent with the welfare comparisons presented in rows 1-3 of Table 3, and thereby generalizes the results of those examples. And from the forms that the welfare functions take, it is clear that if the dynastic planner’s valuation of generation 2-4 utilities simply represent a monotone transform of generation 1’s valuation, then the results of Propositions 1 and 2 carry through directly. Proposition 4 considers a different class of cases in which generation 1 disregards the welfare of generations 3 and 4.

**PROPOSITION 4:** If generation 1’s utility is a homothetic function of its own consumption and the utility of generation 2, generation 2’s utility can be expressed as a homothetic function $u^2\left[c_2, \eta(c_3, c_4)\right]$, dynastic welfare can be expressed as $\psi\left[c_1, c_2, \eta(c_3, c_4)\right]$, and $u^3(c_3, c_4)$ is also a homothetic function, then the Rule against Perpetuities cannot increase dynastic welfare.

**PROOF:** Consider first the case with the Rule against Perpetuities. Since $u^3(c_3, c_4)$ is a homothetic function then generation 3’s utility can be expressed as $z_2 g_3\left(\frac{c_3}{z_2}, \frac{c_4}{z_2}\right)$, in which $g_3\left(\frac{c_3}{z_2}, \frac{c_4}{z_2}\right)$ is a quasi-concave function and $z_2$ is the bequest left by generation 2. It follows that generation 3’s utility-maximizing choices of $\frac{c_3}{z_2}$ and $\frac{c_4}{z_2}$ are independent of the level of $z_2$.

Since $\eta(c_3, c_4)$ is homothetic, it can be expressed as $z_2 h\left(\frac{c_3}{z_1}, \frac{c_4}{z_1}\right)$; and since $u^2\left[c_2, \eta(c_3, c_4)\right]$ is also homothetic, it can be expressed as $z_2 h\left(\frac{c_3}{z_1}, \frac{c_4}{z_1}\right)$. Generation 2’s choices of the shares of its resources devoted to own consumption $\frac{c_2}{z_1}$ and bequests $\frac{z_2}{z_1}$ (subject to $1 \geq \frac{c_2}{z_1} + \frac{z_2}{z_1}$) are therefore independent of $z_1$. The same is obviously also true of generation 1’s choices of $\frac{c_2}{z_1}, \frac{c_3}{z_1}$ and $\frac{c_4}{z_1}$ in the absence of the Rule against Perpetuities.
Consequently, the first order condition characterizing generation 1’s choice of its own consumption level is:

$$\frac{\partial u^1}{\partial c_1} = h \left[ \frac{c_2}{z_1}, \frac{z_2}{z_1} \eta^* \left( \frac{c_3}{z_2}, \frac{c_4}{z_2} \right) \right].$$

The right side of (17) is smaller with the Rule against Perpetuities than without, since the Rule imposes restrictions on generation 1’s choices. Hence the left side of (17) must be smaller with the Rule against Perpetuities than without, from which the quasi-convexity of generation 1’s utility function, together with its homotheticity, implies that that imposition of the Rule cannot be accompanied by both a reduction in first generation consumption and a rise in second generation utility. Since second generation utility cannot rise unless first generation consumption declines, it follows that second generation utility cannot rise with imposition of the Rule.

The first order condition characterizing generation 2’s consumption choice is:

$$\frac{\partial u^2}{\partial c_2} = \eta^* \left( \frac{c_3}{z_2}, \frac{c_4}{z_2} \right).$$

Since \(\eta^* \left( \frac{c_3}{z_2}, \frac{c_4}{z_2} \right)\) is smaller with the Rule against Perpetuities than without, it follows from the quasi-convexity of generation 2’s utility function, together with its homotheticity, that imposition of the Rule cannot simultaneously reduce \(c_2\) and increase \(\eta\). And since \(c_2\) must decline in order for \(\eta\) to rise, it follows that \(\eta\) must decline.

Consequently, imposition of the Rule against Perpetuities does not increase any argument of the dynastic welfare function. Both the utility of the second generation and \(\eta\) decline, and the utility of generation 1 cannot rise with the imposition of this constraint on its choice. Since the dynastic welfare function is Paretian, and no argument increases, its value cannot rise.

Proposition 4 demonstrates that if generation 1 disregards the utilities of generations 3 and 4 (except insofar as they influence the utility of generation 2), generation 2 values generation 3 and 4 utilities in the same way that the dynastic planner does, and generations have homothetic preferences, then the Rule against Perpetuities cannot improve dynastic welfare. The mechanism that produces this result is the misallocation between generations 3 and 4 that arises when generation 3 has the discretion afforded by the Rule against Perpetuities. This misallocation makes generation 2 disinclined to leave a large bequest, and has the same effect on generation 1. The exercise in section 3, the results of which are displayed in rows 4-6 of Table 3, is one example of the phenomenon captured by Proposition 4.
5. **Implications**

The analysis in sections 2-4 suggests that the Rule against Perpetuities reduces dynastic welfare in a broad range of cases.

Recent legislative developments have seriously reduced the scope of the Rule against Perpetuities in the United States. As of 2014, 18 states have replaced the Rule with much looser restrictions, in some cases permitting grantors to establish perpetual trusts, in others permitting the formation of trusts lasting 360 or even 1,000 years. Jurisdictional competition is commonly cited as the cause of this development (Sitkoff and Schanzenbach, 2005; Schanzenbach and Sitkoff, 2006).

6. **Conclusion**

The Rule against Perpetuities limits the range of options available to those seeking to transfer resources to future generations. One of the concerns in imposing such a legal restriction is that individuals with fewer options may decide to reduce their transfers and instead spend more on themselves; this is problematic if, as is likely, transfers are insufficient to maximize dynastic welfare in the absence of restrictions. And even if wealthy individuals do not increase their own consumption levels in response to the restrictions imposed by the Rule against Perpetuity, their inability to limit the consumption opportunities of the next generation typically produces outcomes that are worse for the dynasty.

If individuals behave rationally and value the relative welfares of future generations in the same way that a dynastic planner would, then there is only a very restricted set of cases in which imposition of the Rule against Perpetuities can improve dynastic welfare. This leaves two potentially broad classes of cases that could favor the Rule, the first of which is that individuals behave irrationally in seeking to transfer resources to distant generations, and that a prohibition on this behavior leads to better outcomes. It is entirely possible, perhaps likely, that some intergenerational transfers have irrational aspects – from which, to be sure, it does not directly
follow that the Rule against Perpetuities would improve matters, since the welfare impact of the Rule then depends on the precise nature of the irrationality and its implications for behavior. Advocates commonly argue that attempted transfers to distant generations are often the product of irrational feelings of omniscience and omnipotence, which if true, and systematic, suggests that the Rule might serve a useful corrective role.

A second potential justification for the Rule is that it is not in society’s interest to permit dynasties to maximize their own welfares. This argument relies on the notion that the pursuit of dynastic welfare imposes a negative externality on others, perhaps by affecting the nature of social or political relationships. This possibility, while real, is ruled out by the standard Paretian welfare functions used in this analysis; and it might be noted that inconsistency between the objectives of families and the objectives of the society at large carry implications that go way beyond the Rule against Perpetuities. The origins of the Rule, its traditional justifications, have nothing to do with a desire to impose costs on property-owning families for the betterment of society. Instead the Rule was thought to help families help themselves; but it appears that, despite the inconsistency between individual and dynastic decision making, the Rule against Perpetuities is apt to make things worse rather than better.
References


Farhi, Emmanuel and Ivan Werning, Progressive estate taxation, Quarterly Journal of Economics, May 2010, 125 (2), 635-673.


Leach, W. Barton and Owen Tudor, The Rule against Perpetuities (Boston: Little, Brown, 1957).


Tate, Joshua C., Perpetual trusts and the settlor’s intent, Kansas Law Review, 2005, 53, 595-626.

Table 1

Effect of the Rule against Perpetuities on Consumption and Welfare

Panel A: Consumption

<table>
<thead>
<tr>
<th>Allocation</th>
<th>Generation 1</th>
<th>Generation 2</th>
<th>Generation 3</th>
<th>Generation 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule against Perpetuities</td>
<td>$\frac{1}{8}y$</td>
<td>$\frac{7}{32}y$</td>
<td>$\frac{21}{64}y$</td>
<td>$\frac{21}{64}y$</td>
</tr>
<tr>
<td>No Rule against Perpetuities</td>
<td>$\frac{1}{8}y$</td>
<td>$\frac{1}{8}y$</td>
<td>$\frac{1}{4}y$</td>
<td>$\frac{1}{2}y$</td>
</tr>
<tr>
<td>Dynastic Welfare Max</td>
<td>$\frac{1}{15}y$</td>
<td>$\frac{2}{15}y$</td>
<td>$\frac{4}{15}y$</td>
<td>$\frac{8}{15}y$</td>
</tr>
</tbody>
</table>

Panel B: Welfare Levels

<table>
<thead>
<tr>
<th>Allocation</th>
<th>Generation 1</th>
<th>Generation 2</th>
<th>Generation 3</th>
<th>Generation 4</th>
<th>Dynasty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule against Perpetuities</td>
<td>-10.29</td>
<td>-4.86</td>
<td>-2.23</td>
<td>-1.11</td>
<td>-18.49</td>
</tr>
<tr>
<td></td>
<td>+ 8 ln(y)</td>
<td>+ 4 ln(y)</td>
<td>+ 2 ln(y)</td>
<td>+ ln(y)</td>
<td>+ 15 ln(y)</td>
</tr>
<tr>
<td>No Rule against Perpetuities</td>
<td>-9.70</td>
<td>-4.85</td>
<td>-2.08</td>
<td>-0.69</td>
<td>-17.32</td>
</tr>
<tr>
<td></td>
<td>+ 8 ln(y)</td>
<td>+ 4 ln(y)</td>
<td>+ 2 ln(y)</td>
<td>+ ln(y)</td>
<td>+ 15 ln(y)</td>
</tr>
<tr>
<td>Dynastic Welfare Max</td>
<td>-9.88</td>
<td>-4.59</td>
<td>-1.95</td>
<td>-0.63</td>
<td>-17.05</td>
</tr>
<tr>
<td></td>
<td>+ 8 ln(y)</td>
<td>+ 4 ln(y)</td>
<td>+ 2 ln(y)</td>
<td>+ ln(y)</td>
<td>+ 15 ln(y)</td>
</tr>
</tbody>
</table>

Note: Table 1 presents outcomes for an example with four generations and in which individual utility equals the sum of the utilities of all future generations plus the natural log of own consumption. Dynastic welfare is the sum of the utilities of all four generations. Panel A presents consumption levels given the aggregate resource limit $y$; Panel B presents utility levels of each generation and aggregate dynastic welfare. The first row of each panel presents outcomes if the Rule against Perpetuities prevents generation-skipping transfers. The second row presents outcomes in the absence of the Rule against Perpetuities. And the third row corresponds to consumption choices that maximize dynastic welfare.
Table 2

Effect of the Rule against Perpetuities with Intergenerational Discounting

Panel A: Consumption

<table>
<thead>
<tr>
<th>Allocation</th>
<th>Generation 1</th>
<th>Generation 2</th>
<th>Generation 3</th>
<th>Generation 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule against Perpetuities</td>
<td>$\frac{2}{5}y$</td>
<td>$\frac{6}{25}y$</td>
<td>$\frac{6}{25}y$</td>
<td>$\frac{3}{25}y$</td>
</tr>
<tr>
<td>No Rule against Perpetuities</td>
<td>$\frac{2}{5}y$</td>
<td>$\frac{1}{5}y$</td>
<td>$\frac{1}{5}y$</td>
<td>$\frac{1}{5}y$</td>
</tr>
<tr>
<td>Dynastic Welfare Max</td>
<td>$\frac{1}{4}y$</td>
<td>$\frac{1}{4}y$</td>
<td>$\frac{1}{4}y$</td>
<td>$\frac{1}{4}y$</td>
</tr>
</tbody>
</table>

Panel B: Welfare Levels

<table>
<thead>
<tr>
<th>Allocation</th>
<th>Generation 1</th>
<th>Generation 2</th>
<th>Generation 3</th>
<th>Generation 4</th>
<th>Dynasty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule against Perpetuities</td>
<td>-3.40 + 2.5 ln(y)</td>
<td>-3.20 + 2 ln(y)</td>
<td>-2.49 + 1.5 ln(y)</td>
<td>-2.12 + ln(y)</td>
<td>-5.98</td>
</tr>
<tr>
<td>No Rule against Perpetuities</td>
<td>-3.33 + 2.5 ln(y)</td>
<td>-3.22 + 2 ln(y)</td>
<td>-2.41 + 1.5 ln(y)</td>
<td>-1.61 + ln(y)</td>
<td>-5.74</td>
</tr>
<tr>
<td>Dynastic Welfare Max</td>
<td>-3.47 + 2.5 ln(y)</td>
<td>-2.77 + 2 ln(y)</td>
<td>-2.08 + 1.5 ln(y)</td>
<td>-1.39 + ln(y)</td>
<td>-5.54</td>
</tr>
</tbody>
</table>

Note: Table 2 presents outcomes for an example with four generations and in which individual utility equals the discounted sum of the utilities of all future generations plus the natural log of own consumption. The generational discount factor is 0.5, so a succeeding generation receives a welfare weight of 0.5, the subsequent generation receives a welfare weight of 0.25, and the following generation receives a welfare weight of 0.125. Dynastic welfare is the discounted sum of the utilities of all four generations, using the same generational discount factor of 0.5. Panel A presents consumption levels given the aggregate resource limit $y$; Panel B presents utility levels of each generation and aggregate dynastic welfare. The first row of each panel presents outcomes if the Rule against Perpetuities prevents generation-skipping transfers. The second row presents outcomes in the absence of the Rule against Perpetuities. And the third row corresponds to consumption choices that maximize dynastic welfare.
Table 3

Effect of the Rule against Perpetuities with Idiosyncratic First-Generation Preferences

<table>
<thead>
<tr>
<th>Discount factor $(\alpha)$</th>
<th>Generation 1 preferences</th>
<th>System with greater dynastic welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Generation 2 $(\beta)$</td>
<td>Generation 3 $(\gamma)$</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>1</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: Table 3 presents outcomes for an example with four generations and in which the individual utilities of generations 2-4 equal the discounted sum of the utilities of all future generations plus the natural log of own consumption. Generation 1’s utility is likewise the discounted sum of the utilities of all future generations plus the natural log of own consumption but generation 1 applies a discount factor $\beta$ to generation 2’s utility, a discount factor $\gamma$ to generation 3’s utility, and a discount factor $\delta$ to generation 4’s utility. Dynastic welfare is the discounted sum of the utilities of all four generations, using the same generational discount factor $(\alpha)$ used by generations 2-4 and used by generation 1 in addition to application of its generation-specific discount factors. The first four columns display the values of $\alpha$, $\beta$, $\gamma$ and $\delta$ used in each simulation, and the fifth column indicates whether dynastic welfare is higher with or without the Rule against Perpetuities (“No RAP” denoting cases in which dynastic welfare is greater without the Rule against Perpetuities, and “RAP” denoting cases in which dynastic welfare is greater with the Rule against Perpetuities).