Dear Michigan students, friends and colleagues:

I have enclosed a draft of an evolutionary dynamics project, pitched for a crowd that is relatively comfortable with differential equations. If it’s been awhile since you thought much about calculus, you might find the supplement to be a relatively gentler (though longer) introduction to this work. If you still find the supplement rough going, worry not. My presentation will walk you through the project even more slowly.

Two notes with regard to reading. First, you may want to look at the figures and graphs online, as they are in color. Second, the last part of the supplement includes robustness checks that are for an older version of the model: we are in the process of re-running those.

Looking forward to our discussion soon!

Best,
Daria
CAT AND MOUSE: THE EVOLUTIONARY DYNAMICS OF GETTING AROUND THE LAW

Daria Roithmayr, Justin Chin, Fei Fang and Bruce Levin

Abstract

Regulators in many fields of law—tax shelters, payday lending, intellectual property—play a costly game of cat and mouse with those who innovate to escape regulation. Payday lenders, for example, have developed a series of innovative loopholes to avoid regulation, each more ingenious than the last; most recently, lenders have partnered with Indian tribes to claim immunity from state regulation altogether. Insight into this ever-evolving game of innovation comes from an unexpected place: recent evolutionary dynamics work in biology on adaptive drug resistance. Math models have used coupled differential equations to help public health officials optimize the timing and strength of drug treatment. Borrowing from these models, we develop a simple model to describe the dynamic “cat and mouse” relationship, with an eye towards advising regulators about how to optimize their strategy under constrained resources. We counsel regulators that in many instances, the optimal strategy surprisingly is to respond quite slowly to the latest innovative loophole or to weaken the strength of regulation. These counterintuitive results show that the evolutionary dynamics of biological systems can potentially shed light on the dynamics of social systems, as actors innovatively adapt to each other in new and often unpredictable ways.
In many fields of law, actors who are regulated are constantly hunting for new ways to escape regulation. Taxpayers are perpetually creating new tax shelters. Intellectual property pirates are always inventing new versions of file-sharing technology that aren’t covered by the regulation. High frequency traders write new and innovative algorithms that actually use regulation to bait the trap for unsuspecting institutional traders. In response to these innovators, regulators must continually return to the drawing board to address the latest innovation, a process that is costly and wasteful. The cat-and-mouse problem emerges most often in fields in which the financial stakes are high and in which hackers can innovate with the stroke of a pen.\(^1\)

This regulatory arms race plays a central role in payday lending. Over the last two decades, predatory payday lenders have generated new and increasingly creative strategies to escape the reach of state (and now federal) regulation. Payday lenders’ latest ingenious innovation has been to partner with Indian tribes to be able to claim immunity from all state regulation.\(^2\) In turn, regulators have had to enact new regulation to re-target the lenders, which then triggers further lender innovation, and so on.

In this paper, we investigate the dynamics of this game between regulators and those they regulate, with an eye towards giving advice to the regulator. Insight comes from an unexpected place: recent work in biology on adaptive resistance. Biologists have modeled the cat and mouse game between adaptive pathogens on one side and the human immune system or drug treatments on the other. These models use coupled differential equations to describe the co-evolving relationship between pathogens that mutate, and immune systems that mutate in response, triggering further mutation in the pathogens and so on. Such models have helped public health officials to optimize their “regulatory” strategy against continually evolving pathogens.\(^3\)

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1 For example, tax shelter promoters can create a new shelter merely by drafting a new legal partnership. Payday lenders can draft a simple legal agreement with an Indian tribe. (Buell, Katz). In HFT, traders have baited front-running traps using the SEC rule requiring rerouting of orders to exchanges with the lowest price. Peter Henning, High Frequency Trading Falls Through the Cracks of Criminal Law, The New York Times, April 7, 2014. For digital piracy, see Minku Kang, Justin Chin and Brandon Hartstein, Digital Copyright Piracy and Enforcement: An Arms Race Model (unpublished, on file with the authors). Innovative workarounds are very common in high frequency trading, tax compliance, cybersecurity and spam regulation, and intellectual property regulation.


Drawing on this literature, we develop a simple mathematical model to investigate this regulatory arms race. As in biological models, we use coupled differential equations to describe the reciprocal influence that law exerts on economic behavior, and in turn, that behavior exerts on law. We then conduct numerical simulations to optimize the timing and strength of a regulator’s strategy in the face of ever-evolving legal resistance. More specifically, we counsel the regulator about how to allocate resources towards minimizing both predatory lending and the need to re-regulate.

Based on our toy model, we make two counterintuitive suggestions to the regulator. First, a regulator who cares as much about minimizing prohibited behavior as about the need to constantly retarget its regulation should slow the rate at which it adapts to escaping innovators. Second, a regulator who faces budget constraints that make minimizing costly re-regulation a top priority can do so by weakening its regulations. As these surprising prescriptions suggest, dynamic models of biological systems can potentially shed light on the hard-to-forecast dynamics of social systems, as actors innovatively adapt to each other in new and often unpredictable ways.

**Co-evolutionary Model of Relationships Between Regulation and Regulated Actors**

We develop a probabilistic model of coupled differential equations to describe one regulator interacting with multiple regulated actors. Regulated actors continually innovate to escape each new regulation, and the regulator must continually innovate to re-regulate the escaping innovators.

The history of payday lending legislation in Maryland illustrates the arms race dynamics that we model and the deadweight costs such an arms race imposes. In the late 1990s, lenders unsuccessfully tried to obtain an exemption from Maryland's usury limit on interest rates, but consumer lenders quashed those efforts. In response, lenders began partnering with out of state and then national banks to be able to claim the interest rate of the bank’s home state. Maryland legislators countered with regulation making each of these moves illegal.

Lenders then tried a series of innovations, each countered by retargeting regulation: adding exorbitant service fees on the loans, offering "tax refund anticipation loans,” and most recently, partnering with a federally recognized Indian tribe to claim immunity from all state regulation. Federal regulators have now stepped in to regulate at the national level. To stay one step ahead of lenders, federal officials now closely monitor consumer complaints with the goal of mounting a

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4 The history of the Maryland payday lending arms race is chronicled at length in Gomez v. Jackson Hewitt, 427 Md. 128 46 A.3d 443 (2011).


6 Courts have yet to rule on the validity of the "rent-a-tribe” strategy: a Maryland federal court recently ruled that it would be more appropriate for the Maryland Commission on Financial Regulation to determine whether a tribal partnership entitled lenders to avoid state regulation. The parties then settled shortly after the federal court's ruling but before the commission could take up the question. See Adam Mayle, Usury on the Reservation: Regulation of Tribal-Affiliated Payday Lenders, 31 Rev. of Banking & Fin. Law 1053 (2011). See also Western Sky Financial v. Maryland Comm’r of Fin. Reg., 2012 WL 1284407 (unpublished). See Adam Mayle, Usury on the Reservation: Regulation of Tribal-Affiliated Payday Lenders, 31 Rev. of Banking & Fin. Law 1053 (2011).
response as quickly as possible. They also respond with the strongest regulation possible. But, as we suggest below, a quick and strong response might actually be counterproductive, for the same reason that immediate and high-strength antibiotic treatment might actually encourage drug resistance.

The dynamic model that we propose borrows from a recent literature in biology that uses coupled differential equations to map the evolutionary dynamics of drug resistance in bacteria. These equations describe the dynamic evolution over time of bacteria and their “regulators:” viruses that are genetically engineered using the bacteria’s own DNA to attack the bacteria. The mathematical model we propose here is similar to predator-prey models and is a variation of the population dynamics of lytic phage and bacteria found in (1). More details are available in the Appendix SI.

We model a coupled equation system of multiple lenders and a single state regulator. We denote a population of strategy-specific predatory lenders of generation $i$ as $L_i$, where $L_i$ is the number of lenders in a strategy-specific population. The subscript $i$ represents the particular strategy that lenders pursue (and a matching subscript for the regulator represents the matching state regulation that can effectively regulate this generational strategy.)

The value of $L$ changes over time, as ordinary business forces affect the market. The population of lenders grows logistically at rate $g$ until it reaches market saturation level $K$, and lenders exit the market at rate $f$ owing to business failure. In addition, the market loses lenders as the state regulator closes predatory lenders down (at lender closure rate $d$).

The population dynamics equation that represents the evolving density of lenders is thus:

$$\frac{dL_i}{dt} = L_i \cdot g \cdot \left(1 - \frac{L_i}{K}\right) - (L_i \cdot f) - (L_i \cdot R_i \cdot d) \tag{1}$$

In Equation 1, the first term on the right in brackets represents logistic lender growth. The second term in parentheses represents lender failure for business reasons, and the third term models the closure or “death” of lenders from regulatory attack.

In Equation 2 below, regulation matures over time in tandem with the lender population.

$$\frac{dR_i}{dt} = R_i \cdot m_i \cdot \left(1 - \frac{R_i}{1}\right) \tag{2}$$

Here, the term $R_i$ represents a single strategy-specific regulator (state or federal). Because we have only a single regulator, the number of regulators does not change over time as it does for lenders. Rather, $R_i$ depicts the evolution of the regulation’s maturity from 0 to 1, as it progresses.

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7 CFPB conversation.


9 $L_T$ represents the total number of lenders in the population, consisting of all strategies/generations.
from early glimmer of an idea through to committee report or proposed regulation, and finally to well-settled regulation fully clarified by jurisprudence. The maturation value of \( R \), ranging from 0 to 1, reflects this life cycle of regulation, and a more mature regulation can close lenders more efficiently, as reflected in Equation 1.

Regulation matures logistically at a rate \( m \). More specifically, a single generation of strategy specific regulation of type \( i \) matures logistically at rate \( m \) towards full maturity at an \( R_i \) of 1. Thus, Equation 2 represents the dynamics of a strategy-specific regulator “developmentally” maturing from 0 to 1.

The system of lenders and a state regulator described by the above two equations converges to a steady state of lenders under regulatory pressure by a mature regulation, represented by \( L_i = (1 - (d + f)/g) \). The derivation of this equilibrium point is provided in the Appendix SI.

We now extend the model to include the evolution, via innovative behavior, by both lenders and the state regulator. Lenders “mutate” or innovate with some probability to discover strategies that enable them to escape the regulator. The regulator’s ability to close down lenders creates selection pressure that favors innovative predatory lenders, who outcompete lenders using the older strategy. Put more formally, in proportionate response to lender closure rates, lenders discover and adopt with some probability an innovative strategy that enables them to escape regulation.

Introducing this new “mutant” lender strategy into the model is fairly straightforward. Proportionate to the closure of lenders, some fraction of innovative lenders from the previous generation \( i \)-\( l \) will develop a new mutant innovative strategy \( i \) to escape regulation. Likewise, some fraction of lenders will copy those innovators. Pursuant to innovation and copying, populations of generation \( i \) will switch to a next-generation innovative strategy, \( i + l \), at rate \( (Q_{i,i+1}) \). Thus, each generation \( i \) loses lenders as those lenders escape to the next-generation strategy \( i + l \). In addition, each generation also gains lenders from the previous generation, as they switch to the instant strategy \( i \) from an earlier, outdated strategy \( i - l \).

The equation describing the evolution of lenders via innovation over time is modified to include lender innovation, which is described as a sort of mutation:

\[
\frac{dL_i}{dt} = \left[ L_i \ast g \ast \left( 1 - \frac{L_i}{K} \right) \right] - \left( L_i \ast f \right) - \left( L_i \ast R_i \ast d \right) + \left[ L_{i-1} \ast \left( Q_{i-1,i} \right) \right] - \left( L_i \ast \left( Q_{i,i+1} \right) \right)
\]

In Equation 3, innovative lenders are added from the last generation and also lost to the next generation, as evolution continues. More specifically, the fourth term (in brackets) represents this addition of mutant lenders from the previous generation and the subtraction of mutant lenders from the instant generation who have innovated to join the subsequent generation.

In response to lender innovation, the regulator adaptively innovates as well. In particular, the regulator discovers with some probability \( p \) an innovative strategy to re-regulate the escaping lenders. Here, \( p \) represents the resources and effort that the regulator devotes to innovating to discover a new regulatory strategy. The regulator’s probability of discovering how to re-regulate is a stochastic term, and is proportional to the relative fraction of escape lenders, \( \frac{L_i}{L_T} \), as well as the maturity of the existing regulation \( R_i \). We thus represent the probability of the actual event of adaptation, adjusted for changing conditions and for uncertainty, as \( P_{i \rightarrow i+1} \). This term is a random variable drawn from a distribution described in more detail in the SI/Appendix.
The modified dynamics for the regulator, which now includes the birth of a new regulation, is written thus:

\[
\frac{dR_i}{dt} = P_{i-1,t} + m \times Ri \times \left(1 - \frac{R_i}{R_k}\right)
\]  

[4]

In Equation 4, the first term is the new “birth term” that represents the probability that an earlier-generation regulation will adapt to re-regulate the latest generation of mutant lenders. As before, the second term is the growth or maturation term. In our model, regulation does not regress in maturity. Regulation is additive, meaning each new generation is able to address lenders with a matching subscript and all lenders from earlier generations.

**The Regulator’s Strategy: Strength and Timing**

In managing drug resistance, public health officials must balance the need to eliminate harmful pathogens with the need to minimize the number of new drugs that researchers must discover to control newly-resistant pathogens. Optimizing regulation in the face of evolving lenders poses the same central challenge for the regulator. The regulator must minimize levels of predatory lending at the same time it minimizes the cost of retargeting regulation to effectively “recapture” escaped lenders who have innovated their way out of regulatory reach.

The regulator controls two key parameters: regulatory speed and regulatory strength. First, the regulator decides with what speed to respond to a new lender innovation, represented by the control parameter \(p\).\(^{10}\) This parameter represents the maximum probability in the next time step that the regulator will adapt to re-regulate the escaped lenders, assuming that all lenders have escaped to the next generation. The inverse of this term \(\frac{1}{p}\) specifies the amount of time that the regulator allows to elapse before beginning the process of developing a regulatory response.

Regulators typically control the rate at which they can adapt to lender innovations by hiring more lawyers and legislative assistants to improve the regulator’s ability to retarget regulation to recapture escaping lenders. As we will discuss, budget constraints often impede a regulator’s ability to re-regulate escaped lenders.

Second, the regulator decides how strong to make the regulation, as measured by the rate at which the regulation closes predatory lenders. This parameter is \(d\), the lender death or closure rate, measured in lenders closed per given period of time. Here, regulators can choose whether to adopt very strong regulation, for example, interest rate caps that prohibit lending over a threshold interest rate, or relatively weaker regulation, for example, roll-over prohibitions that limit the number of times a borrower can roll their loan over to a subsequent borrowing period.\(^{11}\) These control parameters affect the rise and fall of payday lender generations, and the emergence and maturation of re-regulation.

\(^{10}\) Importantly, \(p\) is the regulatory control set by the regulator, who chooses for example how many lawyers to assign to the task of re-regulation, as noted above. But adapting is uncertain, and the actual probability of adapting may not correspond to the regulator’s attempts. Accordingly, the model employs \(P\), which is the stochastic version of \(p\), and reflects the actual value of \(P\) during a simulation. Details are in the Appendix.

\(^{11}\) Insert footnote.
Fig. 1: TIME SERIES SHOWING VARIATION OF $p$ ON DYNAMICS

Fig. 1 shows a time series of a single simulation run with four generations (each represented by a color) in which we varied the rate of $p$ holding other parameters constant, to observe the effect of doing so on the number of payday lenders and on the emergence of regulation and re-regulation.

We observe that increasing $p$ facilitates both fewer lenders in a given time period and the earlier emergence of new innovative generations of regulators. A more adaptive regulator is able to close

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For all these and subsequent figures, we conduct simulations using the following baseline parameters. Importantly, these parameters produce the relevant dynamics we discuss herein. In the Appendix/SI, we conduct robustness checks to demonstrate the range of parameters over which we observe these dynamics. It should be noted however, that this dynamic system could also produce
more lenders, but must re-regulate more frequently/earlier in a given period of time. Details about these tradeoffs are included in the SI/Appendix.

**The Optimal Control Problem and Associated Simulations**

We can now run multiple simulations of the model, using the baseline parameters in Table 1 below, with the goal of optimizing the regulator’s performance. We measure regulatory performance by assessing the regulator’s ability to minimize two objective measures: (i) the level of predatory lending, as measured by the average number of lenders in a given time period; and (ii) the frequency with which the regulator must re-regulate to keep up with innovating lenders, as measured by the average frequency of re-regulation in a given time period.

\[
J(\lambda) = \lambda \text{(Average Predatory Lenders)} + (1-\lambda) \text{(Re-regulation innovation frequency)}
\]

\[J(\lambda) = \lambda (\text{avgLT}) + (1-\lambda) (\text{avgRI}) \tag{5}\]

Following the standard “weighted sum” optimization approach, we put both objectives weighted for relative importance into a single objective value function. We also normalize the function to facilitate optimization in unit-less space without regard to measurement scale.

As we show in the Appendix, each parameter forces the regulator to balance the two regulatory objectives against each other. As Fig. 1 illustrates, increasing \( p \), the rate at which the regulator adapts its regulation to re-regulate, reduces predatory lending but also increases (by definition) the frequency with which the regulator must re-regulate. Likewise, increasing \( d \), the regulation’s lender closure rate, means by definition fewer average lenders, but closing predatory lenders spurs those that remain to innovate to find a workaround, which in turn increases retargeting frequency.

The existence of these tradeoffs implies that there will be some optimal combination of the two regulator parameters, \( p \) and \( d \), that simultaneously minimize the average predatory lender numbers and innovative retargeting frequency. The following discussion investigates how to optimize these regulatory parameters.

**Numerical Simulations**

To optimize, we numerically simulate the model to generate and plot the objective value of all varying combinations of the two regulator parameters, \( p \) and \( d \) over the full \( p-d \) space, \([0,1]\). We also conduct our simulations traversing over the full \( \lambda \) space, \([0,1]\) to explore a range of regulator priorities that allocate different weight to different objectives.

The simulations involve inputting two regulatory parameters, \( p \) and \( d \), to generate two objective values, avgLT (average lender number in a given time period) and avgRI (re-regulation frequency in a given time period). This four dimensional system is depicted in two graphs below, the first representing the input of regulatory parameters, and the second representing the output of objective values. The objective values are normalized, to enable comparison across differing units.
The input graph, Fig 2, depicts the full set of all possible input combinations of $p$ and $d$, simulated over the full range of $\lambda$. Fig. 2 provides a precise representation of the domain of the objective function and the relative optimality of the simulation’s outcomes. Pareto-optimal combinations are in black, and white pixels indicate the least optimal combinations; gradations from white to black indicate relative distance between optimal and least optimal.

Fig. 2 depicts the input of the system. In contrast, Fig 3 below depicts the range of objective outcomes that correspond to the full range of parameter combinations sampled above. The Pareto frontier is now highlighted in green. This frontier consists of optimal (minimized) weighted objective value combinations, neither objective of which could be improved without sacrificing optimality in the other objective. We observe that the range is narrow, meaning that many combinations of $p$ and $d$ produce the same Pareto-optimal outcomes.
FIG 3: PARETO FRONTIER

Regulator Advice

Using the toy model, we now demonstrate its potential to advise the regulator about how to optimize regulatory performance. More formally, we will ask the regulator to indicate a particular weighting of the regulator’s relative performance priorities in the objective value function, and then solve the optimal control problem to optimize the regulator’s control settings of $p$ and $d$.

We first ask the regulator to indicate its relative priorities. The optimal combination of $p$ and $d$ will depend on the relative weight that the regulator allocates respectively to minimizing prohibited behavior and minimizing the frequency of re-regulation. Accordingly, the regulator must first choose a lambda to specify relative weight of each regulatory objective. A lambda of zero indicates that the regulator exclusively prioritizes reducing prohibited behavior; a regulator who theoretically cares only about reducing re-regulation will choose a lambda of one.

A regulator’s choices about relative priorities are likely to vary with the kind of legal regulation. In certain areas of law, for example in tax law, regulators may care almost exclusively about reducing prohibited behavior. This is because the cost of (re) regulating is small relative to the revenues saved by reducing tax avoidance, and smaller than in many other areas of law because the IRS can often re-regulate to disallow innovative avoidance in more informal ways that do not
require legislating. In other areas, like payday lending, the benefits of reducing prohibited behavior might be more speculative, given the uncertain welfare effects of predatory lending. Indeed, there are those who argue that some level of payday lending is welfare-promoting. The cost of re-regulating may also be relatively higher because lenders frequently change identities (to become credit service organizations, or to make title loans) that allow them to escape altogether the jurisdiction of the regulator. As a result, payday lending regulators might choose a lambda closer to .5 than the lambda chosen by tax regulators.

Once the regulator has chosen a lambda that reflects its relative priorities, we can use Figs. 2 and 3 and their underlying data to identify the optimal \( p \) and \( d \) combination for which the regulator should strive. For example, if a regulator cares equally about both objectives, choosing a lambda of .5 (in the middle of the foil in Fig. 3), then we can identify the regulator’s optimal \( p, d \) combination as \( p = .04 \) and \( d = .52 \). (We can also observe this \( p, d \) combination in Fig. 2. This optimal combination consists of the regulator choosing a very low probability of innovating in a given time, and a moderate strength of regulation. This is a surprising result that we will discuss in detail below.

Our analysis also enables us to advise the regulator about what optimal regulatory performance entails. For example, the corresponding optimal outcomes associated with the optimal \( p, d \) combination corresponding to \( \lambda = 0.5 \) are as follows: a re-regulation frequency of a new regulation approximately every three years (.04 per month) and an optimal average lender number of 500 lenders per month.

Perhaps most importantly, we can also identify the optimal timing for re-regulation, meaning the amount of time from a new lender innovation to the first effort to re-regulate. That value is given by the inverse of the re-regulation rate, \( 1/p \): here, the optimal delay would be twenty four months (1/.04) before a regulator began any effort to re-regulate escaping lenders.

Armed with this information, the regulator can make adjustments to current regulatory controls in order to optimize regulatory performance. For example, on the basis of the information above, if a regulator had been trying to respond as quickly as possible to escaping lender innovation, the regulator would adjust by dramatically slowing its response to reach the identified optimal rate \( p \) of .04.

Finally, our analysis enables us (and policymakers) to observe the hard-to-predict dynamic relationships between the regulator’s controls and regulatory performance. Figs. 4a and b depict the way in which the regulator’s optimal choices vary with the regulator’s relative priorities. Here, we observe two surprising results. First, as the regulator’s priority moves more towards an equal weighting of objectives, the regulator’s optimal \( p \) drops dramatically. As noted earlier, for a lambda of .5, the regulators optimal rate of \( p \) is .08. Second, as the regulator moves away from an equal weighting or priorities, and moves towards the exclusive priority of reducing re-regulation, the regulator’s optimal \( p \) rises again, precipitously and unexpectedly. One would expect that as reducing re-regulation becomes more of a priority, a regulator should reduce the probability with which it innovates to re-regulate. But surprisingly, Fig. 4a shows exactly the opposite!

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Why might a regulator want to drop its rate of adaptation so dramatically, and then raise it again? We can best understand these results by describing the role that $p$ and $d$ in achieving each regulatory objective. Imagine two bins of lenders, a “first generation” and a “next generation” bin, the latter containing lender who have mutated to innovate around regulation in the first-generation bin. The parameter $p$ controls the “chase” rate: how quickly the regulator can move from first generation to next generation in order to chase escaping lenders. A higher $p$ means a quicker regulator jump into the next generation bin to chase escaping lenders. The parameter $d$ controls two features: the death or “kill” rate of lenders in each bin once the regulator has entered the bin, and correspondingly, the pressure that lenders feel (or more formally, their selection pressure) to innovatively escape regulation into the next generation bin.

When the regulator exclusively prioritizes reducing lender numbers, at $\lambda = 0$, the regulator’s optimal $p$ and $d$ are both at their maximum. In this region, the optimal regulatory strategy is to chase and kill lenders as aggressively as the regulator can, without regard to how frequently the regulator has to re-target escaping lenders. It is less important that lenders have a very strong incentive to escape by innovating (owing to the high closure rate) because the regulator is at maximum $d$ and $p$ working as aggressively as possible to track down and close those escape lenders.

As regulator objectives approach equal weighting, with a $\lambda=0.5$, and the regulator becomes more concerned with reducing re-regulation, the regulator’s optimal strategy changes quickly. In this region, the optimal strategy is to dramatically reduce the probability that the regulator will
discover a new regulatory strategy to address escaping lenders. Of note, $p$ drops farther and faster than one might predict. As Fig. 4a shows, the optimal $p$ drops to a level of .10 as lambda nears .4.

As lambda continues to drop, the optimal strategy now changes to a second strategy. This strategy consists of a dramatically low $d$ and a high $p$, which we refer to as a “kill them slowly enough that they never notice” strategy. This strategy works by eliminating the regulated actors so slowly that they have no real incentive to innovate—they are in no hurry to innovate, because no alarm is raised, so to speak, by a low $d$. They do not perceive the regulatory threat to be high. At the same time, a high $p$ (very adaptive regulator) enables the regulators to chase down the few who do innovate to adopt a strategy that enables them to escape.

This “kill them slowly strategy,” which helps to control lender numbers while lender numbers remain a priority, is confirmed by the regulator’s optimal strategy as lambda nears 1. This lambda constitutes a mythical regulator who has no interest in reducing prohibited behavior in any way, and cares only about reducing or avoiding the cost of re-regulation. Accordingly, the regulator’s optimal strategy is now to reduce $p$ and reduce $d$ to their minimum, essentially to give up regulation entirely. In sum, the regulator can progressively improve its ability to reduce re-regulation through two strategies, first by first dramatically reducing its adaptability (intuitive) but then by weakening the strength of regulation to kill them slowly (counterintuitive).

It bears repeating that these estimates (and the above discussion) are hypothetical estimates based on artificially derived parameters in a toy model, without benefit of observational data or validation of the model. This discussion is meant merely to illustrate the potential capacity of the model to provide information with which to advise a regulator. A more precise estimate of the relevant times and regulation strength would depend on development of first, a plausible model derived from first principles with well-estimated parameters, and subsequently, a model that has been validated with observed data.

V. DISCUSSION

Regulators face a fundamental challenge in designing and enforcing regulation: how to manage ongoing innovation to avoid compliance. Existing legal literature offers very little in the way of a theoretical framework to understand the arms race of innovation between regulators and those they regulate, and no frameworks model the endogenous change that innovation generates over time.\textsuperscript{14}

Such frameworks are potentially of immense importance; practical remedies for innovative regulatory hackers are few and far between. Some legal scholars have recommended the adoption of general anti-abuse rules meant to penalize attempts to innovate around legal regulation. One version of these rules penalizes innovative workarounds if they are undertaken in bad faith for the sole purpose of avoiding regulation.\textsuperscript{15} In tax, this approach has been codified in the so-called

\textsuperscript{14} One study has proposed optimizing the timing of tax reform to minimize wasteful regulatory innovation, using a static model that bears a resemblance to our model. (Hines 2004). This study suggests that the optimal timing of tax reform balances the tradeoff between the benefits of slower re-regulation in reducing the incentive for taxpayers to innovate, and the costs of that regulation in allowing tax avoidance to take place with less restriction. This model does not depict or take into account time or endogenous innovation.

\textsuperscript{15} Scholars have suggested that policymakers target the bad intent of regulatory hackers or design regulation broad enough to shut down all potential loopholes, but policymakers have never adopted such
“economic substance doctrine.” Another version of these rules in tax disallows workarounds if they are inconsistent with the legislature’s intentions in enacting the particular rule on which the innovator bases her workaround.16

Legal practitioners find these remedies incomplete, in part because regulatory hackers can avoid significant tax obligations even during the relatively short window of time that it takes for the IRS or a court to declare that a shelter violates the economic substance doctrine or is inconsistent with legislative intent.17 These general rules require courts to decide whether they apply to a particular transaction: they are not self-executing or determinable by some list of objective criteria.18 In Canada, for example, applying a general rule involves various stages with burdens of proof that shift at each stage, all of which can prove time consuming.19 In addition, such general rules are vulnerable to arguments that they are too vague or stifle desirable innovation. In general, no proposed remedy has yet proved wholly effective, in tax or in other areas like payday lending in which cat and mouse games are a problem.20

Where the cat-and-mouse problem of innovative workaround is poorly studied in legal scholarship, the problem is well-described and understood in the field of biological resistance. Biologists have for some decades developed mathematical models that describe the emergence of resistant pathogens that evolve in response to immune system regulation or regulation by drug treatment. Public health officials have used these models to design effective strategies to manage and prevent the emergence of pathogenic resistance.

Just as immediate treatment with a strong antibiotic can encourage the emergence of drug resistance, our model reflects a world in which immediate efforts to re-regulate escaping taxpayers or payday lenders encourage yet more innovation by the escapers. This toy model paves the way for the potential development of a plausible model in which the relevant parameters can be estimated, and ultimately, the model fit to observable data regarding the periodic rise and fall of innovations to escape the law.

regulation, likely because it risks limiting legitimate business innovation and because it is vulnerable to being struck down as overly vague (Buell, Katz). But see Stopping SPAM, Goodman, Study of Email Spam, Wang, Irani and Pu.


17 Interview with Ed Kleinbard, Former Chief of Staff of U.S. Congress’ Joint Committee on Taxation, March 2017.

18 Cesare Silvani, GAARs in Developing Countries, IFA Research Paper 70, 74 (2013).

19 In Canadian GAAR litigation, the Minister of National Revenue (“Minister”) must first allege that a transaction is an ‘avoidance transaction.’ The taxpayer then has the burden to prove otherwise on the basis that the transaction can “reasonably be considered to have been undertaken or arranged primarily for bona fide purposes other than to obtain the tax benefit.” (Income Tax Act 1985 S.245(3)(a)). If the taxpayer fails to prove that the transaction is not an ‘avoidance transaction’ then the Minister has the burden of proving, on a balance of probabilities, that the avoidance transaction would result in an ‘abuse and misuse’ (either directly or indirectly) of a provision in any of the instruments specified in s.245(4), which includes the ITA. Avi-Yonah and Pichhadze, infra, p. 10.

20 Interview with Nanette Davis, US Department of Justice, Criminal Tax Division, January 22, 2017.
These models capture an important aspect of the real world of policymaking in the law, in which endogenous innovation pushes relationships between regulator and regulated out of equilibrium, and in which actors play for as much time as they can get from their most recent move. These models appear potentially useful in the world of finance, where the cost of innovation can be low (the creation of a legal relationship with an exempt entity, for example) and the benefit of escaping regulation, even for a short time, quite financially profitable.

More generally, legal theorists should consider making use of evolutionary dynamics models developed in the context of biological systems. Probabilistic models might be particularly useful in describing the behavior of agents navigating in uncertain environments, whether those agents are pathogens, animal species or humans. Such models have proved useful in other contexts; for example, models of contagion have been deployed to assess the likelihood of violence among humans or the spread of information among actors. Biological models might well provide legal scholars with useful ways of describing the way in which actors innovatively adapt to each other in new and often unpredictable ways.

VI. CONCLUSION

This paper investigates the cat-and-mouse game of regulatory avoidance, with an eye towards advising regulators about how best to manage actors who are continually evolving to avoid regulation. Borrowing from models of pathogen resistance, we develop a simple mathematical model to describe the dynamics of innovation and re-regulation between regulators and those they regulate. Simulating the toy model, we provide advice to the regulator on an optimal “response time” to innovative loophole entrepreneurs, and an optimal regulator strength as well. This toy model describes a key mutualistic “arms race” relationship between regulator and regulated. We develop this model in large part to argue for theoretical work that endogenizes innovation as a key factor driving legal (and behavioral) change over time.

More broadly, we hope our work will spur further theoretical study of the co-evolutionary relationship between regulation and behavior. Such work is particularly important in areas in which innovation happens at a very rapid pace, and may have important implications for automated regulation using artificial intelligence (“smart” red lights are only one such example.) Further inquiry can help us to understand how best to design and optimize regulatory institutions that appropriately take account of the way in which regulation and behavior mutually shape each other.
APPENDIX

Parameter | Significance
--- | ---
dt | Time step (month)
L<sub>i</sub> | Population of lenders of order i (number of lenders)
R<sub>j</sub> | Regulator pursuing strategy j (R \( \in [0,1] \), where 0 = immature; 1 = mature.)
K | Lender carrying capacity (lenders)
g | Lender natural growth rate (lenders/month)
f | Lender natural failure rate (lenders/month)
d | Lender closure rate resulting from regulatory strategy i (lenders/month)
s | Maximum “mutation” rate of lender experimental innovation (under ideal conditions of high lender closure)
cc | Maximum probability of lender copying behavior under ideal conditions
q | Maximum probability of lender experimental innovation (s+cc)
f | Adjusted probability of lender copying mutant lenders (responding to changing conditions)
\( Q_{i,i+1} \) | Actual lender adaptation probability from \( L_i \) to \( L_{i+1} \) (in given time step)—incorporating Monte Carlo protocol
p | Maximum probability of regulator innovation and development (in given time step) under ideal conditions of high lender escape
e | Adjusted probability of regulator innovation under changing conditions
\( P_{i,i+1} \) | Actual regulator adaptation probability from \( R_j \) to \( R_{j+1} \) (in given time step)—incorporating Monte Carlo protocol

1.0 THE MODEL OVERVIEW

Supp. Fig. 1
2.0 Literature on Arms Races and Optimizing

We develop a model that closely resembles the standard co-evolutionary model biologists use to map the arms race relationship between mutating pathogens and their adversaries. Each side in the arms race exerts selective pressure on the other that favors “mutants” able to recapture the adversarial advantage.\(^1\) A small literature captures the use of population dynamics models to optimize the strategy of one or both sides in the arms race.\(^2\) As discussed more fully in the text, our analysis draws on similar techniques to help regulators map the kind of co-evolutionary tradeoffs we see in drug resistance, in which more adaptive regulators reap significant benefits but also serve to trigger additional innovations allowing regulated actors to escape the reach of regulation.

In the security and computer industries, the problem has gotten a fair amount of attention. Scholars have used dynamic or “differential game” analysis together to better understand the adversarial arms races between hackers and security firms in market competition (Spiegel and Tookes 2013; Bandyopadhyay et al. 2014; Frei et al. 2009).\(^3\) SPAM has also been modeled in a similar way. (Goodman, Heckerman and Routwaite 2005).

3.0 Mapping the Probabilistic Relationship Between Regulator and Lenders

Lenders and a single state regulator are engaged in an arms race, in which each side adopts an innovative mutant strategy to respond to the other’s latest move. Lenders adopt an innovative


\(^{22}\) Population dynamics models have done much to enable public health officials to optimize their regulatory approach. For example, researchers have used population dynamics models to set the timing of drug treatment in order to minimize the evolution of resistance. Public health officials can generate schedules to interrupt treatment (to allow the drug-sensitive incumbents a chance to outcompete the mutants) or to switch or sequence a series of treatments to manage mutant pathogens that have escaped “regulation.” Ryan Zurakowski and Andrew Teel, A model predictive control based scheduling method for HIV therapy, 237 J. Theor. Biol. 368 (2006); Rutao Luo, Michael J. Piovoso, Javier Martinez-Picado, and Ryan Zurakowski, Optimal Antiviral Switching to Minimize Resistance Risk in HIV Therapy, 6(11) PLoS ONE e27047 (2011). More recently, researchers have used principles of population dynamics to develop a “time machine,” a sort of crop rotation sequencing of drugs that enables them to cycle pathogens back to a non-resistant, “wild-type” state. Robert E. Beardmore and Rafael Pena-Miller, Rotating antibiotics selects optimally against resistance in theory, 7(3) Math. Biosciences and Engin. 527 (2010). But see Sebastian Bonhoeffer, Pia Abel zur Wiesch and Roger D. Kouyos, Rotating antibiotics does not select optimally against resistance, 7(4) Math. Biosciences and Engin. 919 (2010). Economists and finance scholars have modeled the arms race between firms innovating in order to steal each others’ consumers (“the new Pepsi”), with an eye toward optimizing the firm level of investment in such innovation. Matthew Spiegel and Heather Tookes, Dynamic Competition, Valuation and Merger Activity, 68 J. Fin. 125 (2013) (firms engaged in an innovation arms race with each other to steal customers).

\(^{23}\) See, e.g, Matthew Spiegel and Heather Tookes, Dynamic Competition, Valuation and Merger Activity, 68 J. Fin. 125 (2013) (firms engaged in an innovation arms race with each other to steal customers); Tridib Bandyopadhyay, Dengpan Liu, Vijay Mookerjee, and Allen Wilhite, Dynamic Competition in IT Security: A Differential Games Approach, 16(4) Inform. Syst. Frontier 643 (2014) (same-industry firms engaged in an arms race with innovative hackers and with each other, the latter to avoid being the most tempting hacker target).
mutant strategy to avoid regulation. The model describes behavior probabilistically to reflect the uncertainty associated with these dynamic processes.

3.1 Lenders

Lenders are strategy specific. That is, lenders possess identical incentives, market volume, and behavior, and differ only in the lending strategy employed. L0 might represent a lender population that employs ordinary payday lending strategies. L1 might represent a lender strategy that has affiliated with Indian tribes to avoid regulation.

Equation 1 represents the change in strategy-specific lender population of type i over time.

$$\frac{dL_i}{dt} = \left[ L_i \cdot g \cdot \left( 1 - \frac{L_i}{K} \right) \right] - \left( L_i \cdot f \right) - \left( L_i \cdot R_i \cdot d \right) + \left[ L_{i-1} \cdot \left( Q_{i-1,i} \right) - L_i \cdot \left( Q_{i, i+1} \right) \right]$$

The first term in brackets represents the addition of lenders to the population via ordinary market growth. The second term in parentheses represents the exit of lenders for ordinary business reasons. The third term shows the exit of lenders who close or are shut down because of regulatory restrictions. The fourth and final term in brackets represents the additions and losses to the population as lenders using an older strategy (i-1) switch to the current strategy (i), and as lenders using the current strategy innovate to adopt a new strategy (i+1) that escapes regulation. Each of these terms is discussed in more detail below.

The first term in the equation represents additions to the lender population from logistic market growth: $L_i \cdot g \cdot \left( 1 - \frac{L_i}{K} \right)$. The variable $g$ represents the lenders’ growth rate (in lenders per month), and $K$ represents the carrying capacity or absolute limit of lenders that the state can support (in lenders). Lenders exit the market as they fail or close for ordinary business reasons; this term is also a function of existing lenders, $L_i \cdot f$. The variable $f$ represents lenders’ business failure rate (in lenders per month).

Lenders also exit the market when they close because of regulatory restrictions, either because state enforcement of the regulation compels them to close or they voluntarily close in response to regulation. The term representing lender regulatory closure is $R_i \cdot d \cdot L_i$. The variable $d$ represents the lender’s closure or death rate from regulation (in lenders per month); this term reflects the strength of the legislation, so for example, stricter legislation like interest rate caps will produce a higher $d$ than less strict legislation like restrictions on the number of times a borrower can roll over a loan. All lenders close with the same probability.

Let $R_i$ represent a single strategy-specific regulator. For example, R0 might represent a regulator that deploys an interest rate cap, where R1 represents a regulator that prohibits affiliation with Indian tribes. Regulation operates by closing lenders as described above.

Regulatory closure depends not just on existing lenders but also on the “developmental stage” of the regulation. Though one often thinks of regulation in a binary form (“on” or “off”), in reality, regulation evolves over time, maturing from white paper to bill to new law to legislation fully developed via judicial opinion and practice. We define the value of $R_i$ to be the maturity level of

---

24 For an analysis of relative strength of payday lending regulation, comparing price caps to other forms of regulation like limits on rollovers or information disclosure requirements, see Creola Johnson, Payday Loans: Shrewd Business or Predatory Lending? 87 Minn. L. Rev. 2002.
the regulation and varies between 0 and 1, with 1 being a fully mature regulation. Even proposed legislation can close lenders, but obviously the more mature regulation is more effective at closing lenders. The concept of regulatory maturity is more fully discussed in the next section.

The last part of the lender equation represents mutation of lenders over time. In the final term of the equation, the strategy-specific lender population using strategy $i$ gains lenders from earlier generations ($i-1$) who switch to the instant strategy. The population also loses lenders who are using the current strategy when they mutate to adopt a next-generation ($i+1$) strategy to avoid regulation.

Lenders switch to a new strategy in one of two ways: by discovering a new strategy that avoids regulation, or (ii) by copying the innovations of other innovative lenders who have already switched. Lender discovery is represented by the term $s \ast R_i \ast d_i$. The variable $s$ (in lenders per month) represents the maximum probability with which lenders innovate to discover a new strategy in a given time period. Lender discovery is proportional both to the maturity of the regulation $R_i$ and the lender regulatory closure rate $d_i$. This proportionality reflects the idea that a high lender closure rate creates significant selection pressure to innovatively adapt, as does a more mature regulation. Thus lender discovery is an adjusted term that reflects changing conditions. Lender copying is represented by the term $cc \ast \frac{L_i}{L_i + L_{i+1}}$. The variable $cc$ represents the maximum rate (in lenders per month) at which lenders copy innovators described above. As the term shows, lender copying is proportional to the fraction of lenders who have switched a next-generation innovation strategy relative to the sum of the current and next-generation lenders. Thus this term is also an adjusted term that reflects changing conditions.

---

25 These two channels of adaptation reflect real-life lender choices that depend on their size. Large national lenders, with offices in many states, adapt by innovating to discover new strategies that avoid the reach of existing regulation. Smaller “mom and pop” lenders adapt by copying those larger lenders. Often, large lenders will announce their latest innovations at the annual Community Financial Services Association meeting, which is the industry’s representative agency, responsible for lobbying state and now the federal government. [Get cite.] We will have more to say about lobbying in a subsequent section.
Lender adaptation is a combination of lender discovery and lender copying. We define \( q_{i,i+1} \) as lender adaptation. This term is the sum of the probability of lender discovery (adjusted to changing conditions) plus lender copying (adjusted to changing conditions). Thus, \( q_{i,i+1} = s \cdot R_i \cdot d_i + c \cdot c \cdot \frac{L_{i+1}}{L_i + L_{i+1}} \).

\( Q_{i,i+1} \) is the stochastic version of \( q_{i,i+1} \). To reflect that adaptation is a stochastic process, we utilize a Monte Carlo protocol to introduce a small amount of randomness in the rate at which lenders adapt through innovation and copycat behavior. We pick a random variable, and if the variable falls within the range of zero to \( q_{i,i+1} \), then lender adaptation occurs with magnitude (must define—rate or % of lenders who adapt) of random variable. \( Q_{i,i+1} \) is the stochastic value—zero if MC fails, or random if MC succeeds. The sum \( q_{i,i+1} \) acts as both the probability of lender adaptation and the upper bound of the lender adaptation rate \( Q_{i,i+1} \) in a given time step. Thus, adaptation from \( i \) to \( i+1 \) is represented by \( L_i \cdot (Q_{i,i+1}) \) where \( 0 \leq Q_{i,i+1} \leq q_{i,i+1} \). For simplicity, we have assumed that lender adaptation is linear and does not skip generations.

Putting all these components together, Equation 1 again summarizes the population dynamics of lenders using a given strategy \( i \):

\[
\frac{dL_i}{dt} = \left[ L_i \cdot g \cdot \left(1 - \frac{L_T}{K}\right)\right] - (L_i \cdot f) - (L_i \cdot R_i \cdot d_i) + [L_{i-1} \cdot (Q_{i-1,i}) - L_i \cdot (Q_{i,i+1})]
\]

In the sections below, we use our model to study a system with four equations representing four generations of lender strategy, each representing a successive next-generation innovation to escape regulation:

\[
\begin{align*}
\frac{dL_0}{dt} &= \left[ L_0 \cdot g \cdot \left(1 - \frac{L_T}{K}\right)\right] - (L_0 \cdot f) - (L_0 \cdot R_0 \cdot d) - L_0 \cdot (Q_{0,1}) \\
\frac{dL_1}{dt} &= \left[ L_1 \cdot g \cdot \left(1 - \frac{L_T}{K}\right)\right] - (L_1 \cdot f) - (L_1 \cdot R_1 \cdot d) + L_0 \cdot (Q_{0,1}) - L_1 \cdot (Q_{1,2}) \\
\frac{dL_2}{dt} &= \left[ L_2 \cdot g \cdot \left(1 - \frac{L_T}{K}\right)\right] - (L_2 \cdot f) - (L_2 \cdot R_2 \cdot d) + L_1 \cdot (Q_{1,2}) - L_2 \cdot (Q_{2,3}) \\
\frac{dL_3}{dt} &= \left[ L_3 \cdot g \cdot \left(1 - \frac{L_T}{K}\right)\right] - (L_3 \cdot f) - (L_3 \cdot R_3 \cdot d) + L_2 \cdot (Q_{2,3})
\end{align*}
\]

Here, we calculate the dynamic steady state for the basic regulator-lenders interaction. The system reaches steady state when the market growth of lenders is equal to the loss of lenders from ordinary business failure and regulation, or when the rate of change in lenders is zero.

In calculating the steady state lender population, we assume that the regulation is fully mature at steady state and thus we set \( R_j = 1 \). Recall from (2) that lenders change over time according to this equation:

\[
\frac{dL}{dt} = [L_i \cdot g \cdot \left(1 - \frac{L_T}{K}\right)] \cdot (L_i \cdot f) - (L_i \cdot R_j \cdot d)
\]

We calculate the steady state lender population \( L \) for a one-generation interaction by setting the rate of change at zero and re-arranging terms as follows:
\[
dL/dt = L \cdot g \cdot \left(1 - \frac{L}{K}\right) - L \cdot f - L \cdot 1 \cdot d = 0
\]

\[L_i = (1 - (d + f)) / g\]

Given our baseline parameters, we observe a steady state lender population of 0.6K, or 480, little more than half of our lender saturation baseline parameter of 800.

### 3.2 The Regulator

The following equation describes the logistic maturation (or logistic growth of regulation) of regulation over time:

\[
\frac{dR_i}{dt} = R_i \cdot m_i \cdot (1 - \frac{R_i}{K_R}).
\]

Here, \(K_R\) is defined as the carrying capacity or steady-state limit of regulation maturity, which is 1, and \(m_i\) is the regulation maturity rate (in units of inverse time.) We assume that in the developmental process, regulation only advances, and never regresses.

As described in the main text, the emergence of innovating lenders creates a selection pressure that favors the emergence or “birth” of new regulation that re-regulates the lenders. To begin, regulators choose some level of re-regulatory adaptation \(p\) to allocate resources and effort towards the process of discovering a new re-regulation strategy. This term represents the maximum probability at which the regulator innovates to adopt a re-regulation strategy in a given time period. This term is associated with the regulator’s control of the rate at which it discovers new strategies to re-regulate escaping lenders: hiring more lawyers to brainstorm for new regulation, or the opportunity cost of shifting lawyers from enforcement to innovation, for example.

The equation \(y_{i-1,i} = p \cdot \frac{L_i}{L_T}\) represents the actual probability of the regulator to innovatively adapt, which is proportional to the density of escaping lenders. This \(y\) term is thus an adjusted term that reflects changing conditions.

Finally, \(P_{i-1,i}\) represents the stochastic version of \(y_{i-1,i}\) in which the next generation of regulation is “seeded” from the previous generation. Again, we use a Monte Carlo protocol to generate a random number, and if that number is less than or equal to \(y_i\), we let \(P_{i-1,i} = .01\). This low \(P\) represents the initial “seeding” of the new generation of regulation, which then matures over time.

More formally, \(P_{i-1,i}\) is a random variable drawn from a distribution \(f(L_i, p \text{ and } L_T)\). Here, \(f(P_{i-1,i} = 0.01 = \text{I}(R_i = 0)\) where \(\text{I}\) is an indicator function that takes value 1 if the statement \(R_i = 0\) is true. Similarly, \(f(P_{i-1,i} = 0 = 1 - y_i \cdot \text{I}(R_i = 0)\).

Putting the seeding or birth of re-regulation strategies and their maturation or growth together, the change in regulatory maturity over time is described in Equation 2:

\[26\] Thus the change in regulation maturity \(\frac{dR_i}{dt}\) only grows toward 1. We also assume that regulation is additive, that is, that the regulator continues to deploy earlier strategies and layers the innovative regulatory strategy on top of the earlier strategies. We assume that the first step of regulatory maturity is innovation, as an agent switches from an old strategy to a new one, and thus we incorporate both growth and adaptation into a single term.
\[
\frac{dR_i}{dt} = P_{i-1, i} + R_i \cdot m_i \cdot (1 - \frac{R_i}{K_R}) 
\]  
(2)

Our model describes four generations of regulations corresponding to four generations of lender strategies, the dynamics of which are represented by the equations below. Since \(K_R = 1\), we simplify to get:

\[
\begin{align*}
\frac{dR_0}{dt} &= P_{N, 0} + R_0 \cdot m_0 \cdot (1 - R_0) \\
\frac{dR_1}{dt} &= P_{0, 1} + R_1 \cdot m_1 \cdot (1 - R_1) \\
\frac{dR_2}{dt} &= P_{1, 2} + R_2 \cdot m_2 \cdot (1 - R_2) \\
\frac{dR_3}{dt} &= P_{2, 3} + R_3 \cdot m_3 \cdot (1 - R_3)
\end{align*}
\]

4.0 Regulatory Tradeoffs.

In this section, we numerically simulate our model to investigate the dynamics of lender and regulator adaptation, and the tradeoffs that accompany such adaptation. Because our goal is to help the regulator optimize its regulatory strategy, we focus our inquiry in this section on the two parameters that are within the scope of the regulator's control: regulator adaptability (\(p\)) and the regulatory strategy's closure rate (\(d\)).

The variable \(p\) is a key parameter in regulatory innovation. State and federal regulators can control how quickly they innovate to find new regulation to respond to lender innovation. For example, to improve adaptability, a regulator could assign more lawyers to explore innovative regulatory strategies. To slow the regulation's rate of adaptation, regulators could refrain from taking any action either to search for innovations or to announce new innovative regulation.

The regulatory strategy closure rate \(d\) is also under the regulator’s control to some degree. To improve the regulatory closure rate, the regulator could adopt a stricter form of regulation like interest rate caps. To reduce the closure rate, the regulator could adopt weaker forms of regulation, like foregoing interest rate caps for restrictions on the number of rollovers a borrower can request.

5.0 Time Series Investigation of Regulatory Adaptability

In the following section, we investigate the effect of varying regulator-controlled parameters on the performance of regulation. In particular, we vary \(p\) and \(d\) to observe the effects of doing so on two performance criteria: the level of predatory lending (measured in total number of lenders denoted as \(LT\)) and the frequency of wasteful innovation by lenders and regulators (measured in

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27 In our model, as in real life, state regulators are able to exert control over the speed or probability with which they innovate to respond to lender behavior. For example, they can assign more lawyers to the problem of responding to lender innovation. Alternatively or in addition, they can find ways to cut the cost of innovating, to increase the probability with which they discover a new strategy. Conversely, if warranted, they can delay action or refuse it altogether.
lender-regulator innovations per month).\textsuperscript{28} For all combinations of parameters, we averaged results over 1000 runs. We use Euler method numerical approximation and arbitrarily chosen baseline parameters to conduct the relevant simulations. A simple version of the model is available online; readers can vary parameters for themselves to observe the effect on lender and regulator dynamics.\textsuperscript{29}

5.1 Varying $p$

First, we vary regulatory adaptability $p$ to observe the effect of doing so on the number of predatory lenders ($LT$) and the frequency with which regulator and lenders innovate ($IR$). A more adaptive regulator with a higher $p$ is faster at (has a higher probability of) enacting in a given time step new retargeting regulation that re-regulates to address the latest lender innovation.

Fig. S4 shows a time series of a single run with four generations in which we varied the rate of $p$ holding other parameters constant, to observe the effect of doing so on regulatory performance. In the graphs measuring the number of lenders over time, on the left side (lender side) of Fig. 4a, we observe that increasing $p$ has the effect of decreasing $LT$. The amplitude of the lender generation curves decreases as the regulator becomes more adaptive. Intuitively, a more adaptive regulator closes more lenders.

In the graphs measuring regulatory formation over time, on the right side (regulator side) of Fig. S4, we observe that increasing $p$ facilitates the earlier emergence of new innovative generations of regulators. In particular, regulator generations appear earlier on the x axis, which represents time. Regulatory formation occurs at a higher rate as well. Likewise, we observe on the left side of Fig. S4 that new innovative generations of lenders emerge earlier as well.

Our base case parameters were $dt=0.02$, $L1=10$, $K=800$, $g=2$, $f=0.4$, $d=0.4$, $p=0.2$, $s=0.1$, $cc=0.1$. In the Appendix, we explore the effects of varying these base parameters.

Available at \url{http://faith.cs.uchicago.edu/~stuart/payday.html}. This applet was created with Elm. All code for this and other approximations will be (but is not yet) available at \url{www.dariarothmayr.com}.\textsuperscript{29}
These graphs also illuminate some other important qualitative aspects of regulatory change over time. On the lender side of Fig. S4, we note that the number of lenders LT rises and falls in regular waves that resemble the dynamics of predator and prey. We also note that after the first generation, subsequent generations of lenders (in response to subsequent regulation) do not rise to the steady-state equilibrium because lenders are spread among multiple generations: the population is mixed.

We also observe that for higher levels of p, the tails of the lender generation curves rise progressively higher for each sequential generation. This is because when p is high relative to the

$dt = .02, L1 = 10, K=800, g =2, f=0.4, d=0.4, s=0.1, cc= 0.1$

**FIG S4: TIME SERIES SHOWING VARIATION OF P ON DYNAMICS**
innovation \((s)\) and copying \((cc)\) rate of lenders, regulators are innovating faster than lenders are escaping, causing lenders to remain in existing generations instead of adapting.

We now investigate with more precision the tradeoffs—the simultaneous costs and benefits—that regulators face when increasing their \(p\) to become more adaptive. Here, we systematically vary the value of \(p\) in increments of \(.005\) on the interval \([0.005, 1]\), and measure the effect of this variation on our two performance criteria: the level of predatory lending in a given time period \((LT)\) and the frequency of costly innovation by lenders and regulator during that same period \((IR)\).

We define the performance criteria as follows. The level of predatory lending \(LT\) is calculated as the sum of the total number of predatory lenders across all three generations \(L_0, L_1, L_2\) divide by the total time period in which regulations \(R_0, R_1, R_2\) are active, to produce a single lender per month value. A regulation is considered active when \(R_i \geq 0\); that is, the first formative step has occurred allowing the regulation to impact the corresponding lender population. We exclude the final generation, \(L_3\) and \(R_3\), to minimize edge effects arising from the fixed number of time steps.

We measure the frequency of wasteful innovation \(RI\) as the number of generations of regulations divided by the total active time period for those regulations calculated in the same manner as above.\(^{30}\) Once again, we consider only three generations of regulation: \(R_0, R_1, R_2\). We use this figure to generate a single innovations-per-month value that represents both lender and regulator innovation.

Figs. S5 on the next page more precisely illustrate the tradeoff that regulators face as they become more adaptive and more able to retarget escaping lenders. Fig. S5 illustrates the downside of increasing regulatory adaptability: becoming more adaptive to respond to lender innovation triggers more regulator and lender innovation, as evidenced by the upward sloping line. Because the regulator retargets and closes more escape lenders, and because wasteful innovation is proportional to lender closure in our model (see above), lenders respond to the greater closure rate by increasingly innovating to escape. In turn, regulators then must respond to innovating lenders by innovating themselves to re-target lenders. In sum, becoming more adaptive triggers more wasteful innovation.

\(^{30}\) For simplicity, from hereon in, we collapse this latter performance criteria into a single “innovation” measure, because each state re-regulation is always accompanied by a lender innovation.)
**FIG S5: EFFECT OF VARYING P ON INNOVATION FREQUENCY**

Fig. S6 shows the positive edge of the double-edged sword: as p increases and the regulator becomes more adaptive, the number of lenders during the given time period diminishes, as evidenced by the downward sloping line. The lender population drops in theory because the regulator can retarget more escape mutant lenders.

**FIG S6: EFFECT OF VARYING P ON PREDATORY LENDING**

Figs. S5 and 6 demonstrate that increasing the regulator’s probability of adaptation produces a double-edged sword: a more adaptive regulator reduces predatory lending during a given time
period but also triggers more lender innovation and more costly regulatory modification during the same time period. This is the tradeoff that we will help the regulator to optimize in Part IV.31

5.2 Varying Lender Closure (d)

We now investigate the effect of varying the strength of regulation—the rate of lender closure $d$—on the dynamics of $LT$ and $RI$. As before, we vary the independent variable $d$ on the interval $[0.01, 1]$ in increments of .01, to observe the effect of this variation on (i) $LT$, the level of predatory lending during a given time period; and (ii) $RI$, the frequency of regulator and lender innovation, calculated during that same time period. As before, we observe the effects of varying $d$ both qualitatively and quantitatively.

Figs. S7 and 8 demonstrate that, as with regulator adaptability, a regulatory increase in $d$ also generates a tradeoff. Increasing $d$ increases wasteful innovation frequency in a given time period. We observe this in Fig S7 in the upward slope of the line. This effect turns on the defined relationship between lender closure and innovation. As more lenders close, lender innovation in a given time period increases, and in turn, the regulator must re-regulate more frequently, both of which are wasteful.

$$dt = .02, L1 = 10, K = 800, g = 2, f = 0.4, d = 0.4, s = 0.1, cc = 0.1$$

FIG S7: THE EFFECT OF VARYING LENDER CLOSURE RATE ON INNOVATION FREQUENCY

At the same time, we observe in Fig. 5b that increasing the lender closure rate produces diminishing levels of predatory lending. This result is again intuitive: a regulation with a higher closure rate will reduce the level of predatory lending for all generations.

31 We conduct extensive sensitivity analysis of this relationship and robustness checks on all parameters in the Appendix.
FIG S8: THE EFFECT OF VARYING LENDER CLOSURE RATE ON PREDATORY LENDING

In sum, increasing both adaptability and regulatory closure rate produces a tradeoff. Increasing regulatory adaptability $p$ diminishes predatory lending but also increases wasteful innovation by lenders and regulator alike. Likewise, increasing regulatory closure rate $d$ also diminishes predatory lending but increases wasteful innovation by lenders and regulator as well. In the next section, we will optimize both parameters, separately and then simultaneously, to minimize both predatory lending and wasteful lender-regulator innovation.

6.0 Optimizing

We want to determine the optimal values of $p$ and $d$ that will simultaneously minimize $LT$, predatory lending, at the same time we minimize $IR$, the frequency with which the regulator and lender must innovate to respond to the other’s latest innovation. We focus first on varying $p$ alone for constant $d$, before varying both $p$ and $d$ simultaneously. More specifically, we traverse region $[0, 1] \times [0, 1]$ in $p-d$ space, holding other parameters constant, to measure the effect of such variation on our two performance criteria. We then compare these performances, to choose the best (Pareto-optimal) regulatory performances and in turn the best (most efficient) control parameters that correspond to the best performances.

Table 1 Variables

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>abbrev</th>
<th>units</th>
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<tbody>
<tr>
<td>Regulator adaptation rate</td>
<td>p</td>
<td>Odds/dt</td>
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<tr>
<td>Regulatory strength</td>
<td>d</td>
<td>Lenders/dt</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg Lenders</td>
<td>AvgLT</td>
</tr>
<tr>
<td>Retargeting Frequency</td>
<td>AvgIR</td>
</tr>
</tbody>
</table>

$dt=.02, L1 = 10, K=800, g =2, f=0.4, d=0.4, s=0.1, cc= 0.1$
Table 2 Simulation Parameter Settings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>variable</th>
<th>units</th>
<th>value</th>
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<td>Time step</td>
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<td>months</td>
<td>.02</td>
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<td>Initial generation of lenders</td>
<td>L1</td>
<td>Lenders</td>
<td>10</td>
</tr>
<tr>
<td>Initial maturity of regulator</td>
<td>R1</td>
<td>Reg mat.</td>
<td>.01</td>
</tr>
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<td>Market saturation</td>
<td>K</td>
<td>Lenders</td>
<td>800</td>
</tr>
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<td>Market growth rate</td>
<td>g</td>
<td>Lenders/dt</td>
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</tr>
<tr>
<td>Market failure rate</td>
<td>f</td>
<td>Lenders/dt</td>
<td>0.4</td>
</tr>
<tr>
<td>Lender closure rate</td>
<td>d</td>
<td>Lenders/dt</td>
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</tr>
<tr>
<td>Lender innovation rate</td>
<td>s</td>
<td>Lenders/dt</td>
<td>0.1</td>
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<tr>
<td>Lender copying rate</td>
<td>cc</td>
<td>Lenders/dt</td>
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### 7.0 Lobbying

Here, we extend the analysis briefly to consider the potential effect of lobbying on the relationship between regulator adaptability and regulatory performance. In particular, we want to explore the potential inhibiting effect of lender lobbying on the regulator’s adaptability, where lobbying strength depends on the number of predatory lenders in a given period of time.

To model the effect of lobbying, we assume that past a particular threshold number of lenders ($L_{\text{thresh}}$), $p$ is dampened proportional to lender density. To represent this dampening effect, we specify a discounting parameter of lobbying effectiveness ($L_{\text{eff}}$) to modify $p$. An $L_{\text{eff}}$ of 0 represents no dampening and 1 represents full dampening. At any specified time step, the lobbying term is given by:

$$LP = L_{\text{eff}} \times \left(1 - \frac{L - L_{\text{thresh}}}{K - L_{\text{thresh}}}\right)$$

The effect of $p$ is dampened by $LP$ to give: $p_{\text{eff}} = p(1 - LP)$. This term is substituted for $p$ in equation (8) in the section above.

Figs. 4e and 4f below show two time series panels for lenders and regulators respectively, each with the lobbying function turned “off” for the top row of panels and “on” for the bottom row of panels. Fig. 4e shows the effect of lobbying on the relationship between $p$ and lender dynamics. When comparing between the two rows in this figure, we observe that at all $p$ levels, lobbying leads to a slightly higher amplitude of lender population peaks for each of the lender strategies, meaning that predatory lending levels increase. We also observe longer periods for the lender cycles, meaning each lender strategy is active for a longer period of time in the presence of lobbying.
This increase in lender population at the peaks and longer active periods for lender strategies occurs presumably because lobbying inhibits the ability of a regulation to become fully formed, and delays the emergence of new regulator generations. This explanation is confirmed by Fig. 4f, which is a time series showing the effect of lobbying on the relationship between p and regulator dynamics.
FIG S10: THE EFFECT OF LOBBYING ON REGULATOR DYNAMICS

Here, we observe that lobbying increases the time between regulator generations, confirming that regulator innovation is slower to emerge in the presence of lobbying.

We note that the effect of lobbying appears relatively limited in our graphs. The small size of the lobbying effects may result from the limited carrying capacity of lenders in the market; under our parameters, the market begins to slow and then limit the growth of lenders, constraining their numbers before lobbying can dampen $p$ more appreciably. Future work will explore the effect of lobbying more fully.

8.0 Robustness

As a preliminary test for robustness, we conducted approximations on the model using more extreme values of the parameters $g$, $f$, $s$, and $cc$ across the range of $p$. We did this to verify stable behavior in the parameter space around our choice of base case values. For each of $p = \{0.2, 0.6, 1.0\}$, parameter values were selected from the sets: $g = \{1, 4, 7\}$, $f = \{0.01, 0.5, 1.0\}$, $s = \{0.01, 0.5, 1.0\}$, and $cc = \{0.01, 0.5, 1.0\}$, with all other parameters held to their base values. From the following lender population and regulator formation versus time graphs, we observed that the nature of the tradeoff between total lender population and regulator innovation frequency is preserved; that is, reductions to the total lender population continued to have corresponding increases to the regulator innovation frequency.

We include further minor observations on the effect of varying particular parameters together with the graphs.
VARYING CC, Lenders

p (.02) and varying cc:

p = .02, cc=.01  p=.02, cc=.05  p=.02, cc=1.0

As cc increases, the amount of time taken for populations to emerge and peak appears to be decreasing in a consistent fashion for both L and R. Additionally, lender populations tend to be higher with high cc.
VARYING CC, Regulator

p (.02) and varying cc:

\[
p = .02, \text{ cc} = .01 \quad p = .02, \text{ cc} = .05 \quad p = .02, \text{ cc} = 1.0
\]

\[
p = .06, \text{ cc} = .01 \quad p = .06, \text{ cc} = .5 \quad p = .06, \text{ cc} = 1.0
\]

p (1.0) and varying cc:

\[
p = 1.0, \text{ cc} = .05 \quad p = 1.0, \text{ cc} = .5 \quad p = 1.0, \text{ cc} = 1.0
\]
VARYING f, Lenders

p (.02) and varying f:
- p = .02, f=.001
- p = .02, f=.05
- p = .02, f=1.0

p (.06) and varying f:
- p = .06, f=.001
- p = .06, f=.05
- p = .06, f=1.0

p (1.0) and varying f:
- p = 1.0, f=.001
- p = 1.0, f=.05
- p = 1.0, f=1.0

As expected, increasing f, the natural failure rate of lenders, causes total lender populations to decrease.
VARYING f, REGULATOR

p (.02) and varying f:

\[ p = .02, f=.001 \quad p=.02, f=.05 \quad p=.02, f=1.0 \]

There appears to be virtually no effect on R as f changes; only changes to p show any significant contribution to regulator formation.
VARYING g, LENDERS

P (.02) and varying g

P=.02, g= 1.0  p=.02, g=4.0  p=.02, g=7.0

P (.06) and varying g

P=.06, g= 1.0  p=.06, g=4.0  p=.06, g=7.0

P (1.0) and varying g

P=1.0, g= 1.0  p=1.0, g=4.0  p=1.0, g=7.0

L populations peak at higher values when g, the natural growth rate, increases, but only up to a certain point, after which increasing g has diminishing returns on affecting L.
VARYING g, REGULATOR

P (.02) and varying g

P=.02, g=1.0  p=.02, g=4.0  p=.02, g=7.0

P (.06) and varying g

P=.06, g=1.0  p=.06, g=4.0  p=.06, g=7.0

P (1.0) and varying g

P=1.0, g=1.0  p=1.0, g=4.0  p=1.0, g=7.0

There appears to be virtually no effect on R as g changes.
As s increases, the behavior observed by lenders is similar to increasing cc, with the exception of more pronounced effects for extremely low s. This is not unexpected, as both s and cc affect the emergence of new lender generations, with s directly controlling the initial appearance of an escape lender.
VARYING \( s \), Regulator

\( p \) (.02) and varying \( s \):

\[ p = .02, s = .01 \quad p = .02, s = .05 \quad p = .02, s = 1.0 \]

\( p \) (.06) and varying \( s \):

\[ p = .06, s = .01 \quad p = .06, s = .5 \quad p = .06, s = 1.0 \]

\( p \) (1.0) and varying \( s \):

\[ p = 1.0, s = .05 \quad p = 1.0, s = .5 \quad p = 1.0, s = 1.0 \]